MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Commander Philip Dumas, R.N., F.R.G.S., H.M.S. 'Duncan,' Mediterranean Squadron ;
Henry Eichbaum, 3 Devonshire Terrace, Ventnor, Isle of Wight ;
Frank Flowers, Map Office, Ginsberg Chambers, Johannesburg, Transvaal, South Africa ; and
Louis George Macrory, M.D., M.B., B.C., B.A., Clifton House, Bridge Road, Battersea, S.W.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

J. D. Bharda, Joint Principal, New High School, Bombay, India (proposed by A. Fowler) ;
Tyson Crawford, Optician, Messrs. Dollond & Co., 35 Ludgate Hill, E.C. (proposed by W. H. Maw) ;
E. Vincent Heward, Hereward Cottage, Westgate-on-Sea (proposed by A. C. D. Crommelin) ;
Eber Jachin Sharpe, ex-Master, Board of Trade, Nautical Surveyor, Newport, Monmouthshire (proposed by M. J. O'Sullivan) ;
E. Spiegel, Banker, 120 Bishopsgate Street Within, E.C. (proposed by R. T. A. Innes) ; and
Philip Edward Vizard, Chief Clerk of the Summons and Order Department, Royal Courts of Justice, Belsize Lodge, Belsize Lane, Hampstead, N.W. (proposed by the Hon. Justice Bruce).

Fifty-six presents were announced as having been received since the last meeting, including amongst others :—

A. Auwers, Neue Reduction der Bradley'schen Beobachtungen, Band 1, presented by Professor Auwers ; Carte photographique du Ciel (23 charts), and Loewy and Puiseux, Atlas photographique de la Lune, fasc. 7, presented by the French Minister of Public Instruction ; Four original negatives of the Moon, taken with the Equatorial coudé of the Paris Observatory, presented by M. Puiseux ; Photograph of Langrenus's map of the Moon, presented by M. Loewy ; Portrait of Sir R. S. Ball (enlarged photograph), presented by Sir R. S. Ball.

Note on Photographs of Comet c 1903 (Borrelly), taken with the 30-inch Reflector at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

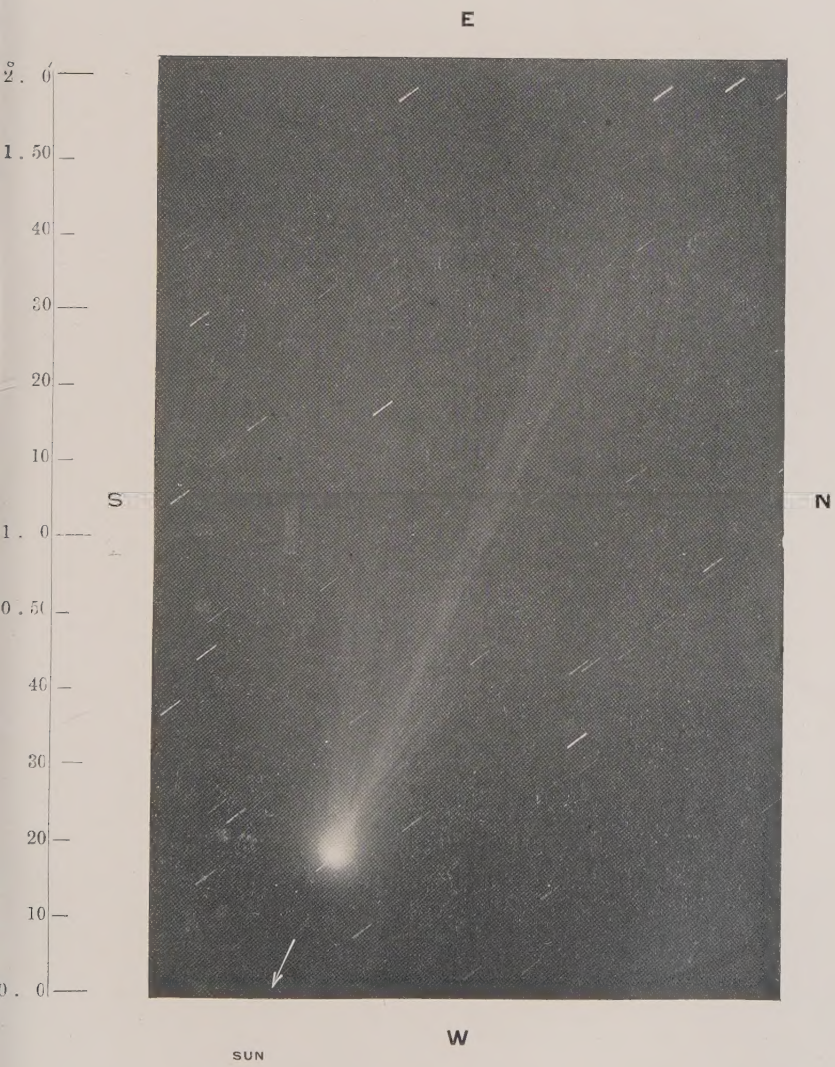
Photographs of this comet for determination of its position have been obtained on twenty-six nights from June 24 to August 14. The instrument was not guided, as the exposures given were short.

The following is a list of the dates on which photographs were obtained and the exposures given :—

Date. 1903.	Exposures.	Date. 1903.	Exposures.
June 24	4^m, 4^m	July 15	30^s, 30^s, 30^s, 30^s, 30^s, 30^s
26	3^m, 3^m, 2^m, 2^m	20	30^s, 30^s, 30^s, 30^s
27	2^m, 2^m, 2^m, 2^m	24	,,
29	2^m, 2^m	26	,,
30	2^m, 2^m, 2^m, 2^m	30	,,
July 1	2^m, 2^m, 2^m, 2^m	Aug. 1	,,
3	1^m, 1^m, 1^m, 1^m	4	,,
6	1^m, 1^m, 1^m, 1^m	5	,,
7	1^m, 1^m, 1^m, 1^m	6	,,
9	1^m, 1^m, 1^m	7	20^s, 20^s, 20^s, 20^s
10	30^s, 30^s, 30^s, 30^s	10	30^s, 30^s, 30^s, 30^s
12	30^s, 30^s, 30^s, 30^s	13	20^s, 20^s, 30^s, 30^s
13	30^s, 30^s, 30^s, 30^s	14	30^s, 30^s, 30^s, 30^s

Photographs with a long exposure, to show the form and structure of the tail, were taken with the 30-inch reflector as follows :—

Date.	Exposure.	Date.	Exposure.
July 20	60^m	July 30	12^m
24	80^m, 30^m	Aug. 1	45^m
26	58^m	4	30^m



COMET 1903 *c* BORRELLY, 1903 AUGUST 1.

PHOTOGRAPHED WITH THE 30-INCH REFLECTOR AT THE ROYAL OBSERVATORY, GREENWICH.

EXPOSURE 45 MINUTES.

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A photograph on a smaller scale was also obtained with the 4-inch Dallmeyer rapid rectilinear lens of 34 inches focus on July 26, exposure 125^m. The most successful photographs with the reflector are the one taken with an exposure of 80^m on July 24 and the one taken on August 1, which is reproduced in Plate 1. The photograph taken on August 1 shows a sharply defined head with nine distinct tails, also sharply defined, the longest extending to the edge of the field, a distance of 125′.

On the Semi-diameter, Parallactic, Inequality, and Variation of the Moon from Greenwich Meridian Observations, 1847·0 to 1901·5. By P. H. Cowell.

In the following paper an account is given of an investigation of the solar parallax from observations of the Moon.

The material used consists of all Greenwich meridian observations with the transit circle from 1847·0 to 1901·5. This interval consists of forty-eight periods of 400 lunar days. Each period of 400 lunar days is treated separately, and 400 lunar days are approximately equal to fifteen anomalistic months and fourteen lunations. Associated, therefore, with any particular value of D, the age of the Moon, there will be found in one period fourteen values of g, the mean anomaly, uniformly distributed round 360°, so that certain classes of error should be to a great extent eliminated in each period.

The tabular places used are based upon Hansen's tables with Newcomb's corrections, and Hansen's error of sign corrected. From 1847–1861 the tabular places are taken from vol. l. of the *Monthly Notices*, and Newcomb's corrections have been applied. From 1862 onwards the tabular places have been taken from the Greenwich annual volumes, and from 1862–1882 Newcomb's corrections and a correction for Hansen's error of sign have been applied.

After this all tabular places received the following corrections:

$$
\begin{aligned}
&+1''\cdot562 \sin D\\
&-0\cdot893 \sin (g+2\varpi-2J)\\
&-0\cdot676 \sin (g+2\varpi+3V-5E)\\
&-0\cdot548 \sin g\\
&+\{N+\Delta g+4''\cdot06 \sin \Omega\}\ 2e \cos g
\end{aligned}
$$

where in the last term N denotes Newcomb's corrections, and is therefore to be taken as zero from 1883 onwards; Δg is a correction to the mean anomaly investigated by myself; its value is $\Delta g = +4''\cdot63-13''\cdot99T+4''\cdot743T^2$ (see *Monthly Notices*,

1903 November) where the rest of the notation and the source of the corrections can be found. It is not necessary to repeat them here.

The present determination is therefore based upon 5647 observations, each observation being compared with a corrected theory, and therefore subject to a less accidental error; for a periodic error in the tabular places is of the nature of an accidental error when terms of a different period are under discussion (unless the argument of the error and the argument under discussion differ by a low multiple of D, the mean elongation). I believe, therefore, that the present discussion is based upon material of greater weight than any previously employed in deducing the solar parallax from lunar observations.

Twice the circumference of the circle, or 720°, has been divided into fifty-seven equal parts; and the observations grouped under the corresponding values of D. The fifty-seventh part of 720° is nearly equal to the movement of D in one lunar day. The difference between the true value of D and the value under which the observation is grouped is not necessarily less than half of $360° \div 57$; for in order to make the grouping mechanical the observations are not always grouped under the nearest value of D in the series of 57. The error may, in fact, amount to 7°, and its average value will be about 2°·5.

The periods of 400 lunar days are numbered from 86 to 133 because period No. 1 begins, with Airy's reductions, on 1750 September 13.

For the sake of greater clearness of explanations I give here the figures for period 86.

TABLE I.

Period 86.

Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.	Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.
	°				°		
0	0·0			12	151·6	−45	5
1	12·6			13	164·2	+ 4	4
2	25·3			14	176·8	− 5	3
3	37·9			15	189·5	+67	3
4	50·5	−12	1	16	202·1	+56	4
5	63·2			17	214·7	+42	1
6	75·8	+11	2	18	227·4	+ 7	2
7	88·4	+ 8	2	19	240·0	−10	1
8	101·0	−30	2	20	252·6	+61	3
9	113·7	+43	6	21	265·3	− 2	5
10	126·3	− 7	6	22	277·9	+35	2
11	139·0	+12	3	23	290·5	−14	4

Ref. No.	Value of D.	Sum of Errors in units of one-tenth of second of arc.	No. of observations.
	°		
24	303·2	−15	2
25	315 8		
26	328·4		
27	341·0		
28	353·7		
29	6·3		
30	19·0		
31	31·6		
32	44·2		
33	56·8		
34	69·5		
35	82·1	− 11	3
36	94·7	+ 14	3
37	107·4	+ 82	3
38	120·0	− 14	3
39	132·6	+ 8	5
40	145·3	+ 33	3
41	157·9	+124	5
42	170·5	− 10	4
43	183·2	+ 43	4
44	195·8	+ 46	2
45	208·4	+ 43	4
46	221·0	+ 29	2
47	233·7	+ 74	4
48	246·3	− 20	2
49	259·0	+ 32	3
50	271·6	+ 3	2
51	284·2	+ 57	4
52	296·8	+ 13	1
53	309·5		
54	322·1		
55	334·7		
56	347·4		

Since the mean error is +6·4 (in units of 0″·1) all errors have been decreased by 6, so as to render this period comparable with other periods. Line 9 therefore reads six errors with a sum +7.

It will be noted that the value of D is proportional to the reference number.

The periods are now arranged in six groups of eight periods each. The following table gives the results for each group and the sums for the whole forty-eight periods, or 54·5 years. The last column but one gives the mean error corresponding to the separate values of D, and the last column gives the same quantities corrected for a change of 0″·45 in semi-diameter, which appears to be indicated by the three preceding columns. Two lines nearest to full Moon are not filled in, as the observations do not always correspond to the same limb.

Table III. that follows gives the values of sin D, cos D, sin 2D, cos 2D, cos 3D, cos 4D for the different values of D: these quantities are required for a least-squares solution. Also in the same table are given the products of some of the above quantities by "n," the number of observations in the group, and E, the sum of the errors of the group.

The sums of lines 0–14, 15–28, 29–42, 43–56 are also taken for certain columns under the titles Σ_1, Σ_2, Σ_3, and Σ_4.

In the last two lines Σ denotes $\Sigma_1+\Sigma_2+\Sigma_3+\Sigma_4$, and Σ' denotes $\Sigma_1-\Sigma_2+\Sigma_3-\Sigma_4$.

TABLE

Periods.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.
	86–93.		94–101.		102–109.		110–117.	
0	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...
3	− 9	1	+ 39	1	...	...	...	...
4	− 59	8	− 54	9	+ 50	3	− 45	4
5	−179	14	−130	8	+ 72	10	+ 50	12
6	− 70	21	−338	15	− 79	17	−184	20
7	− 43	26	−140	25	− 18	19	−142	24
8	− 92	24	−184	31	−103	32	−176	32
9	− 23	31	−110	29	+ 59	28	−243	32
10	−188	30	−170	34	−121	31	+ 28	32
11	−136	25	−213	33	−166	32	+ 36	30
12	−320	30	−305	33	− 74	32	+ 32	28
13	− 6	29	−249	33	− 60	27	+109	34
14	−128	26	−162	30	− 88	36	+ 98	27
15	+238	25	+219	32	− 84	29	+158	30
16	+262	30	+240	33	+119	38	+ 86	24
17	+371	23	+424	31	+ 10	22	+221	26
18	+176	19	+275	21	+100	22	− 51	18
19	+ 38	24	+250	20	+234	28	+ 39	19
20	+ 93	26	+213	17	+ 39	22	− 61	12
21	+125	25	+ 44	23	−135	15	+ 22	13
22	+158	22	+168	27	− 41	16	+ 8	14
23	− 49	18	+243	19	+137	9	+144	15
24	+ 24	10	+ 47	8	+ 17	6	− 66	7
25	...	...	+ 9	1	− 11	1	...	...
26	...	...	...	...	...	...	...	...
27	...	...	...	...	...	...	...	...
28	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...
30	...	...	...	...	...	...	...	...
31	...	...	+ 2	1	...	...	...	...
32	− 53	4	+ 14	1	+ 19	1	...	...
33	−104	7	− 34	6	+ 55	5	− 49	10
34	−117	21	−254	19	+ 66	13	− 69	12
35	−303	17	−214	24	+ 43	25	−139	18

II.

Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Mean in units of o″·01.	Value of ±o″·346 +o″·120 sin D −o″·030 sin2D in units of o″·01.
118–125.		126–133.		86–133.			
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	+ 30	2	...	...
− 2	5	− 67	4	− 177	33	− 54	+41
− 11	8	−207	19	− 405	71	− 57	+43
− 59	23	− 73	18	− 803	114	− 70	+45
+100	22	−139	17	− 382	133	− 27	+46
− 68	28	− 27	29	− 650	176	− 37	+48
+ 60	23	−214	25	− 471	168	− 28	+48
+ 24	17	−105	26	− 532	170	− 31	+47
− 22	29	−199	27	− 700	176	− 40	+46
+ 36	27	−269	29	− 900	179	− 50	+43
− 11	18	−140	32	− 357	173	− 21	+39
− 66	30	−120	36	− 466	185	− 25	+36
+157	28	+303	31	+ 991	175	+ 57	−38
+158	25	+291	33	+1156	183	+ 63	−41
+ 91	20	+252	29	+1369	151	+ 91	−44
+130	22	+ 74	27	+ 704	129	+ 55	−47
+ 16	20	+158	15	+ 735	126	+ 58	−48
− 85	14	+186	15	+ 385	106	+ 36	−48
−150	17	+107	24	+ 13	117	+ 1	−47
+ 76	14	+ 40	13	+ 409	106	+ 39	−46
− 33	16	+ 96	12	+ 538	89	+ 60	−44
+ 39	4	+ 30	6	+ 91	41	+ 22	−42
− 18	1	+ 53	1	+ 33	4	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	+ 2	1	...	...
− 1	3	− 50	4	− 71	13	− 55	+40
−121	9	− 44	13	− 297	50	− 59	+42
− 58	18	−121	17	− 553	100	− 55	+44
−177	15	−144	19	− 934	118	− 79	+46

Periods.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.
	86–93.		94–101.		102–109.		110–117.	
36	−276	23	− 96	24	+ 45	22	−112	20
37	+ 39	22	−240	27	+112	35	− 76	25
38	−430	25	−242	25	−235	38	− 4	26
39	− 83	29	−273	28	−105	28	−121	25
40	−175	27	−328	34	− 60	36	− 11	33
41	− 75	36	−206	30	−156	31	0	21
42	−102	35	−263	27	− 1	35	+104	35
43	+250	35	+ 62	31	+ 20	27	− 91	33
44	+224	37	+323	32	+ 34	27	+129	23
45	+243	28	+143	19	+111	24	+177	28
46	+330	28	+129	31	+236	20	+183	19
47	+196	25	+286	21	− 30	15	+ 6	17
48	+108	26	+178	32	− 71	26	+ 75	21
49	+ 21	28	+212	16	− 47	21	+ 89	15
50	− 77	20	+267	21	+ 83	17	+ 20	17
51	+ 53	16	+198	18	+ 75	20	+ 10	11
52	+111	7	+120	10	+ 25	9	− 67	10
53	+ 38	5	+197	9	...	...	− 4	4
54	+ 28	1	− 13	1	...	...	...	...
55	...	...	...	...	...	...	...	...
56	...	...	...	...	...	...	...	...
Total	+ 29	989	+ 84	1000	+ 76	950	+113	876

TABLE

n.	Ref. No	100 sin D.	*n* sin D.	100 cos D.	*n* cos D.	100 sin 2D.	*n* sin 2D.	100 cos 2D.	*n* cos 2D
	0	0		+100		0		+100	
	1	+22		98		+43		90	
	2	43		90		77		64	
2	3	61	+ 1	79	+ 2	97	+ 2	+ 25	+ 1
33	4	77	25	64	21	98	32	− 19	− 6
71	5	89	63	45	32	81	58	59	42
114	6	97	111	25	29	48	55	88	100
133	7	100	133	+ 3	+ 4	+ 6	+ 8	100	133
176	8	98	172	− 19	−33	−38	−67	93	164
168	9	92	155	40	67	74	124	68	114
170	10	81	138	59	100	95	162	− 30	−51

Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Sum of Errors.	No. of Obs.	Mean in units of 0″·01.	Value of ±0″·346 +0″·120 sin D −0″·030 sin 2D in units of 0″·01.
118–125.		126–133.		86–133			
+ 55	21	−182	25	− 566	135	− 42	+47
+101	27	−207	31	− 271	167	− 16	+48
−130	30	−215	32	−1256	176	− 71	+48
− 47	28	−189	31	− 818	169	− 48	+47
−251	28	−296	38	−1121	196	− 57	+44
+ 16	28	− 29	26	− 450	172	− 26	+41
+187	28	−125	40	− 200	200	− 10	+38
+ 25	30	− 47	31	+ 219	187	+ 12	−36
− 47	25	+278	37	+ 941	181	+ 52	−39
− 38	21	+235	27	+ 871	147	+ 59	−43
− 68	25	+214	32	+1024	155	+ 66	−46
+ 43	17	+232	29	+ 733	124	+ 59	−47
− 44	19	+139	19	+ 385	143	+ 27	−48
+ 9	12	− 11	18	+ 273	110	+ 25	−48
− 4	16	+158	23	+ 447	114	+ 39	−46
+ 42	12	+203	27	+ 581	104	+ 56	−45
− 14	5	+ 82	11	+ 257	52	+ 42	−43
+ 18	2	+ 29	4	+ 278	24	+116	−41
...	...	...	...	+ 15	2	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
−142	830	− 60	1002	+ 100	5647	...	...

III.

100 cos 3D.	n cos 3D.	100 cos 4D.	n cos 4D.	E.	E sin D.	E sin 2D.
+ 100		+ 100		...	...	...
79		+ 64		...	...	...
+ 25		− 19		...	...	...
− 40	− 1	88	− 2	+ 30	+ 18	+ 29
88	29	93	31	− 177	− 136	− 173
99	70	− 30	− 21	− 405	360	328
68	78	+ 55	+ 63	− 803	779	385
− 8	−11	99	132	− 382	382	− 23
+ 55	+97	+ 72	+127	− 650	637	+ 247
95	160	− 8	− 13	− 471	433	349
95	162	82	139	− 532	431	505

n.	Ref. No.	100 sin D.	*n* sin D.	100 cos D.	*n* cos D.	100 sin 2D.	*n* sin 2D.	100 cos 2D.	*n* cos 2D.
176	11	66	116	75	132	99	174	+ 14	+ 25
179	12	48	86	88	158	84	150	55	98
173	13	27	47	96	166	52	90	85	147
185	14	+ 6	+ 11	100	− 185	− 11	− 20	99	+ 183
175	15	− 16	− 28	99	− 173	+ 32	+ 56	95	+ 166
183	16	38	70	93	170	70	128	72	132
151	17	57	86	82	124	94	142	+ 35	+ 53
129	18	74	95	68	88	100	129	− 8	− 10
126	19	87	110	50	63	87	110	50	63
106	20	95	101	30	32	57	60	82	87
117	21	100	117	− 8	− 9	+ 16	+ 19	99	116
106	22	99	105	+ 14	+ 15	− 27	− 29	96	102
89	23	94	84	35	31	66	59	75	67
41	24	84	34	55	23	92	38	− 40	− 16
4	25	70	− 3	72	+ 3	100	− 4	+ 3	0
	26	52		85		89		45	
	27	32		95		61		79	
	28	− 11		99		− 22		98	
	29	+ 11		99		+ 22		98	
	30	32		95		61		79	
1	31	52	+ 1	85	+ 1	89	+ 1	45	0
13	32	70	9	72	9	100	13	+ 3	0
50	33	84	42	55	28	92	46	− 40	− 20
100	34	94	94	35	35	66	66	75	75
118	35	99	117	+ 14	+ 17	+ 27	+ 32	96	113
135	36	100	135	− 8	− 11	− 16	− 22	99	134
167	37	95	159	30	50	57	95	82	137
176	38	87	153	50	88	87	153	50	88
169	39	74	125	68	115	100	169	− 8	− 14
196	40	57	112	82	161	94	184	+ 35	+ 69
172	41	38	65	93	160	70	120	72	124
200	42	+ 16	+ 32	99	− 198	− 32	− 64	95	+ 190
187	43	− 6	− 11	100	− 187	+ 11	+ 21	99	+ 185
181	44	27	49	96	174	52	94	85	154
147	45	48	71	88	129	84	123	55	81
155	46	66	102	75	116	99	153	+ 14	+ 22
124	47	81	100	59	73	95	118	− 30	− 37
143	48	92	132	40	57	74	106	68	97
110	49	98	108	− 19	− 21	+ 38	+ 42	93	102

100 cos 3D.	n cos 3D.	100 cos 4D.	n cos 4D.	E.	E sin D.	E sin 2 D.
+ 55	+ 97	96	169	− 700	462	693
− 8	− 14	− 40	− 72	− 900	432	756
68	118	+ 45	+ 78	− 357	96	186
99	− 183	98	+ 181	− 466	− 28	+ 51
88	− 154	79	+ 138	+ 991	− 159	+ 317
− 40	− 73	+ 3	+ 5	+ 1156	439	809
+ 25	+ 38	− 75	− 113	+ 1369	780	1287
79	102	99	128	+ 704	521	704
100	126	− 50	− 63	+ 735	639	639
79	84	+ 35	+ 37	+ 385	366	219
+ 25	+ 29	95	111	+ 13	13	+ 2
− 40	− 42	85	90	+ 409	405	− 110
88	78	+ 14	+ 12	+ 538	506	355
99	41	− 68	− 28	+ 91	76	84
68	− 3	100	− 4	+ 33	− 23	− 33
− 8		− 59		...	...	...
+ 55		+ 25		...	...	...
95		90		...	...	...
95		90		...	...	...
+ 55		+ 25		...	...	...
− 8	0	− 59	− 1	+ 2	+ 1	+ 2
68	− 9	100	13	− 71	− 50	− 71
99	50	− 68	− 34	− 297	249	273
88	88	+ 14	+ 14	− 553	520	365
− 40	− 47	85	100	− 934	925	− 252
+ 25	+ 34	95	128	− 566	566	+ 91
79	132	+ 35	+ 58	− 271	257	154
100	176	− 50	− 88	− 1256	1093	1093
79	134	99	167	− 818	605	818
+ 25	+ 49	− 75	− 147	− 1121	639	1054
− 40	− 69	+ 3	+ 5	− 450	171	315
88	− 176	79	+ 158	− 200	− 32	+ 64
99	− 185	98	+ 183	+ 219	− 13	+ 24
68	123	+ 45	+ 81	+ 941	254	489
− 8	− 12	− 40	− 59	+ 871	418	732
+ 55	+ 85	96	149	+ 1024	676	1014
95	118	82	102	+ 733	594	696
95	136	− 8	− 11	+ 385	354	285
+ 55	+ 60	+ 72	+ 79	+ 273	268	+ 104

	n.	Ref. No.	100 sin D.	n sin D.	100 cos D.	n cos D.	100 sin 2D.	n sin 2D.	100 cos 2D.	n cos 2D.
	114	50	100	114	+ 3	+ 3	− 6	− 7	100	114
	104	51	97	101	25	26	48	50	88	92
	52	52	89	46	45	23	81	42	59	31
	24	53	77	18	64	15	98	24	− 19	− 5
	2	54	61	− 1	79	+ 2	97	− 2	+ 25	0
		55	43		90		77		+ 64	
		56	−22		+ 98		−43		+ 90	
Σ_1	1580	...	+ 1058		− 753		− 632		−156	
Σ_2	1227	...	− 833		− 587		+ 514		−110	
Σ_3	1497	...	+ 1044		− 693		− 649		−198	
Σ_4	1343	...	− 853		− 688		+ 532		− 36	
		...	...		...		...		...	
Σ	+ 5647	...	+ 416		−2721		− 235		−500	
Σ'	+ 507	...	+ 3788		...		−2327		...	

The hypothesis is now made that E, the sum of the errors in any group, may be equated to

$$n\{\varepsilon \pm \mu + \delta_1 \sin D + \delta_2 \sin 2D\}$$

where

ε represents the mean error ;
μ, the error of semi-diameter, which is here supposed constant ;
δ_1, a correction to the parallactic coefficient ;
δ_2, a correction to the variation coefficient.

Let the observations now be divided into four groups 0–14, 15–28, 29–42, 43–56, and let Σ_1, Σ_2, Σ_3, Σ_4 denote summations over these four groups ; then the normal equations are :

$$5647\varepsilon + 507\mu + (\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4)(\delta_1 \sin D + \delta_2 \sin 2D) = +100$$
$$507\varepsilon + 5647\mu + (\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4)(\delta_1 \sin D + \delta_2 \sin 2D) = -24796$$
$$\varepsilon(\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4) \sin D + \mu(\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4) \sin D + \delta_1 \Sigma \sin^2 D + \delta_2 \Sigma \sin D \sin 2D = -17231$$

and

$$\varepsilon(\Sigma_1 + \Sigma_2 + \Sigma_3 + \Sigma_4) \sin 2D + \mu(\Sigma_1 - \Sigma_2 + \Sigma_3 - \Sigma_4) \sin 2D + \delta_1 \Sigma \sin D \sin 2D + \delta_2 \Sigma \sin^2 2D = +10475$$

The calculations of Table III., remembering that

$$\sin^2 D = \tfrac{1}{2}(1 - \cos 2D)$$
$$\sin^2 2D = \tfrac{1}{2}(1 - \cos 4D)$$
$$\sin D \sin 2D = \tfrac{1}{2}(\cos D - \cos 3D)$$

100 cos 3D.	n cos 3D.	100 cos 4D.	n cos 4D.	E.	E sin D.	E sin 2D.
− 8	− 9	99	113	+ 447	447	− 27
68	71	+ 55	+ 57	+ 581	564	279
99	51	− 30	− 16	+ 257	229	208
88	21	93	22	+ 278	214	272
− 40	− 1	88	− 2	+ 15	− 9	− 15
+ 25		− 19		...	...	...
+ 79		+ 64		...	...	...
+ 12		+ 134		− 5813	− 4158	+ 1907
− 12		+ 57		+ 6424	− 3927	+ 3395
+ 86		+ 13		− 6535	− 5106	+ 2630
− 74		+ 152		+ 6024	− 4040	+ 2543
...		...		...	...	...
+ 12		+ 356		+ 100	− 17231	+ 10475
...		...		− 24796	...	...

enable us to write the normal equations in the numerical form

$$\begin{aligned} 5647\epsilon + 507\mu + 416\delta_1 - 235\delta_2 &= +100 \\ 507\epsilon + 5647\mu + 3788\delta_1 - 2327\delta_2 &= -24796 \\ 416\epsilon + 3788\mu + 3074\delta_1 - 1366\delta_2 &= -17231 \\ -235\epsilon - 2327\mu - 1366\delta_1 + 2645\delta_2 &= +10475 \end{aligned}$$

We do not want the value of ϵ, and it is clear that we may drop the first equation and the terms containing ϵ in the others.

Solving the three equations that remain, I obtain

$$\begin{aligned} -\mu &= +0''\cdot346 \\ +\delta_1 &= -0''\cdot120 \\ +\delta_2 &= +0''\cdot030 \end{aligned}$$

or the tabular semi-diameter requires a correction

$$-0''\cdot346$$

The tabular parallactic coefficient requires a correction

$$+1''\cdot562 + 0''\cdot120$$

(of which the first term had been applied to the individual observations before grouping).

The tabular variation coefficient requires a correction

$$-0''\cdot030$$

About this method of treatment the following points are to be remarked :

(1) The observations of line 14 are not all first-limb observations, nor those of line 43 all second-limb. It has not been thought worth while to undertake the labour of applying the necessary correction. Probably not more than fifty observations, or less than 1 per cent., have been assigned to the wrong limb.

(2) The hypothesis that the error of semi-diameter is a constant is the best that I can make. It will be noticed that the observations grouped in line 7, for example, contain some daylight and some night observations, according to the season of the year. I am not concerned with any difference in the correction applicable to the individual observations of this group, but merely to the mean correction. The mean correction for different ages of the Moon is no doubt a function of the age of the Moon and not a constant. If there were any means of obtaining its numerical values they could be analysed into the form $f(D)+\alpha \sin D+\beta \sin 2D$, when α, β would be corrections applicable to the results of this paper. As far as I can see the difficulty is inherent in this method of finding the solar parallax, which is rendered uncertain by an amount $\frac{1}{14}\alpha$. I do not think that α can possibly be as large as $0''\cdot30$, and it is possibly a good deal smaller, so that the parallax deduced in this paper should not be as much as $0''\cdot02$ in error.

The weights of the values found for $\mu \cdot \delta_1$ and δ_2 are 781, 515,* and 1614 times the weight of a single observation.

The pure error of observation has been found from the transit circle and altazimuth to have a probable amount $0''\cdot75$. But we have to deal here with the probable discordance from the tables as corrected for the terms set out at the beginning of this paper. Perhaps $1''\cdot5$ is a fair estimate. If so, the probable error of this determination of the parallactic coefficient is

$$\pm 0''\cdot07$$

and of the resulting solar parallax $\pm 0''\cdot005$.

Having now completed the investigation of the correction required by Hansen's parallactic terms, it remains to deduce the value of the solar parallax.

Newcomb (*Astronomical Constants*, p. 190) gives the following equation :

$$\text{Parallactic Coefficient} = -124''\cdot66 \frac{\text{Solar Parallax}}{8''\cdot790}$$

This equation assumes the mass of the Moon to be $\frac{1}{81\cdot15}$

* Implying that owing to the difficulty of separating the error of parallactic coefficient from the error of semi-diameter 5647 observations are worth as much as 1030 observations would be worth if the difficulty did not exist.

times that of the Earth ; and as Newcomb considers the denominator of this fraction uncertain by only $\pm 0''.15$ (p. 189), this introduces an inappreciable uncertainty into the value of the solar parallax.

In deducing this equation, however, Newcomb assumes that Hansen's theory is correct, whereas he has himself thrown doubt upon this very point. I have transformed Brown's results (*Memoirs R.A.S.* vol. liii. pp. 89 and 96), and I find that they confirm Delaunay's theory with Newcomb's estimate of the correction on account of terms of high order not calculated by Delaunay, and Hansen is therefore $0''.55$ in error. I therefore obtain the above equation in the revised form :

$$\text{Parallactic Coefficient} = -125''.2 \frac{\text{Solar Parallax}}{8''.790}$$

Hansen's tables and Hansen's theory differ by $0''.033$ in the coefficient of the parallactic term for the same value of the solar parallax ; and Hansen's theory leads to a coefficient of $-122''.032$ (Newcomb, *Astron. Papers Amer. Eph.* vol. i.), and Hansen's tables consequently to

$$-122''.065 \times 1.03573 = -126''.43$$

The coefficient with which the observations have been compared is therefore

$$-126''.43 + 1''.56 = -124''.87$$

and the further correction deduced in this paper alters this to

$$-124''.87 + 0''.120 = -124''.75$$

The solar parallax is therefore

$$8''.76$$

The correction to the mean semi-diameter used since 1847 is $-0''.346$. The semi-diameter corresponding to a parallax of $57'\ 2''.3$ used since 1856 is $15'\ 34''.10$. This quantity should therefore be about $15'\ 33''.75$.

Lastly Hansen's theory, Delaunay and Hill agree as to the coefficient of the variation, but in his tables Hansen has added $0''.34$ to this coefficient to correct for an assumed difference between the Moon's centre of figure and centre of gravity. This paper confirms the coefficient of Hansen's tables. This suggests that the evection should be analysed to see whether it supports Hansen's assumption, and also it appears advisable to transform Hansen's tables in the same way that Newcomb has already transformed his theory.

It will be also noticed that if the semi-diameter and variation are determined from other sources, and values differing by $\Delta\mu$ and $\Delta\delta_2$ from the results of this paper are found for them, then

if $\Delta\delta_1$ and $\Delta\pi$ be the corrections to the parallactic coefficient and solar parallax

$$\Delta\pi = +\frac{1}{14{\cdot}2}\Delta\delta_1 = \frac{1}{14{\cdot}2}\left\{-\frac{3788}{3074}\Delta\mu + \frac{1366}{3074}\Delta\delta_2\right\}$$
$$= -\frac{1}{11{\cdot}5}\Delta\mu + \frac{1}{32{\cdot}0}\Delta\delta_2$$

It should be mentioned that Newcomb in 1876 attributed the apparent increase of the variation to systematic errors of observation varying with the age of the Moon.

On Oscillating Satellites. (Second Paper.)
By H. C. Plummer, M.A.

1. In the earlier paper (*Monthly Notices*, vol. lxiii. p. 436), to which the present is a continuation, periodic orbits which are possible in the neighbourhood of centres of libration have been investigated with a degree of approximation corresponding to the second order of the relative coordinates. The results appear to suggest that a still closer approximation may be made with some advantage, although the discussion relates only to a small part of the restricted problem of three bodies, and has apparently no direct practical bearing. The notation of the former paper will be followed generally, but some changes will be admitted. The three collinear centres of libration alone will be considered here.

2. The general equations of motion of the satellite are taken in the form

$$\ddot{x} - 2\dot{y} = \Omega_{10}, \qquad \ddot{y} + 2\dot{x} = \Omega_{01}$$

where

$$2\Omega = \mu\left(r_1^2 + \frac{2}{r_1}\right) + \nu\left(r_2^2 + \frac{2}{r_2}\right), \quad \Omega_{rs} = \frac{\partial^{r+s}\Omega}{\partial x^r \partial y^s}.$$

Now at the collinear centres of libration $y = 0$ and $\Omega_{10} = \Omega_{01} = 0$; and $\Omega_{rs} = 0$ when s is odd. Hence in relative coordinates the equations of motion are

$$\left.\begin{aligned} \ddot{\xi} - 2\dot{\eta} &= \Omega_{20}\xi + \tfrac{1}{2}(\Omega_{30}\xi^2 + \Omega_{12}\eta^2) + \tfrac{1}{6}(\Omega_{40}\xi^3 + 3\Omega_{22}\xi\eta^2) + \ldots \\ \ddot{\eta} + 2\dot{\xi} &= \Omega_{02}\eta + \Omega_{12}\xi\eta \qquad\qquad + \tfrac{1}{6}(3\Omega_{22}\xi^2\eta + \Omega_{04}\eta^3) + \ldots \end{aligned}\right\} \ldots \quad (1)$$

so long as the satellite remains within the region over which the expansions are possible. It is known that periodic solutions of these equations exist, and these can be appropriately expressed by Fourier series of the form

$$\left.\begin{aligned} \xi &= a_0 + a_1\cos mt + a_2\cos 2mt + a_3\cos 3mt + \ldots \\ \eta &= \quad b_1\sin mt + b_2\sin 2mt + b_3\sin 3mt + \ldots \end{aligned}\right\} \ldots \quad (2)$$

the orbits being symmetrical with respect to the ξ-axis.

If these expressions for ξ and η be substituted in (1) and powers and products of the circular functions be expressed in terms of circular functions of multiples of mt, there will result an identity between two cosine series and an identity between two sine series. Now it is known that a_1, b_1 are of the first order, and a_0, a_2, b_2 of the second order. The order of the other coefficients then corresponds to their suffixes. For if the series (2) be supposed to terminate at the argument imt, the series obtained by substitution in (1) must terminate at the same argument; since it is easy to see that the argument $i'mt$ ($i'>i$) can only arise in connexion with coefficients of order higher than i. Hence the two identities will give $(2i+1)$ equations involving the $(2i+1)$ coefficients and m; and these equations, together with the Jacobian integral, will suffice to furnish a solution to the order of approximation considered.

3. The immediate object is to find a solution which shall be valid to the third order. The expansions of (1) and (2) may therefore be supposed to end with the terms actually written down. Then when (2) are substituted in (1) the following seven equations are obtained by neglecting terms of higher order than the third, and equating the coefficients of the sines and cosines of the different multiples of mt:

$$0 = \Omega_{20}a_0 + \tfrac{1}{4}\Omega_{30}a_1^2 + \tfrac{1}{4}\Omega_{12}b_1^2 \quad \dots \quad \dots \quad \dots \quad (3)$$
$$0 = (m^2+\Omega_{20})a_1 + 2mb_1 + \mathrm{L} \quad \dots \quad \dots \quad \dots \quad (4)$$
$$0 = (m^2+\Omega_{02})b_1 + 2ma_1 + \mathrm{M} \quad \dots \quad \dots \quad \dots \quad (5)$$
$$0 = (4m^2+\Omega_{20})a_2 + 4mb_2 + \tfrac{1}{4}\Omega_{30}a_1^2 - \tfrac{1}{4}\Omega_{12}b_1^2 \quad \dots \quad (6)$$
$$0 = (4m^2+\Omega_{02})b_2 + 4ma_2 + \tfrac{1}{2}\Omega_{12}a_1b_1 \quad \dots \quad \dots \quad (7)$$
$$0 = (9m^2+\Omega_{20})a_3 + 6mb_3 + \mathrm{A}_3 \quad \dots \quad \dots \quad \dots \quad (8)$$
$$0 = (9m^2+\Omega_{02})b_3 + 6ma_3 + \mathrm{B}_3 \quad \dots \quad \dots \quad \dots \quad (9)$$

The quantities L, M, A_3, B_3 are of the third order and represent the expressions

$$\mathrm{L} = \tfrac{1}{2}\Omega_{30}a_1(2a_0+a_2) + \tfrac{1}{2}\Omega_{12}b_1b_2 + \tfrac{1}{8}\Omega_{40}a_1^3 + \tfrac{1}{8}\Omega_{22}a_1b_1^2 \quad \dots \quad (4')$$
$$\mathrm{M} = \tfrac{1}{2}\Omega_{12}(2a_0b_1 - a_2b_1 + a_1b_2) + \tfrac{1}{8}\Omega_{22}a_1^2b_1 + \tfrac{1}{8}\Omega_{04}b_1^3 \quad \dots \quad (5')$$
$$\mathrm{A}_3 = \tfrac{1}{2}\Omega_{30}a_1a_2 - \tfrac{1}{2}\Omega_{12}b_1b_2 + \tfrac{1}{24}\Omega_{40}a_1^3 - \tfrac{1}{8}\Omega_{22}a_1b_1^2 \quad \dots \quad (8')$$
$$\mathrm{B}_3 = \tfrac{1}{2}\Omega_{12}(a_1b_2 + a_2b_1) + \tfrac{1}{8}\Omega_{22}a_1^2b_1 - \tfrac{1}{24}\Omega_{04}b_1^3 \quad \dots \quad (9')$$

4. An eighth equation is required, and this can be obtained from the Jacobian integral. Now

$$\dot{\xi} = -ma_1 \sin mt - 2ma_2 \sin 2mt - 3ma_3 \sin 3mt$$
$$\dot{\eta} = \quad mb_1 \cos mt + 2mb_2 \cos 2mt + 3mb_3 \cos 3mt$$

and

$$2\Omega = 2\Omega_0 + \Omega_{20}\xi^2 + \Omega_{02}\eta^2 + \tfrac{1}{3}\Omega_{30}\xi^3 + \Omega_{12}\xi\eta^2 + \tfrac{1}{12}\Omega_{40}\xi^4 + \tfrac{1}{2}\Omega_{22}\xi^2\eta^2 + \tfrac{1}{12}\Omega_{04}\eta^4$$

In substituting in the equation

$$V^2 = C_0 - C + 2(\Omega - \Omega_0)$$

terms of the fourth order are to be retained, and there results an equation involving cosines of the first four multiples of mt. Their coefficients must clearly vanish, since the integral must be satisfied throughout the motion, and the four relations thus obtained must evidently be involved implicitly in the equations (3)–(9). There will thus remain an equation involving the energy constant which takes this form

$$0 = C_0 - C + \tfrac{1}{2}a_1^2(\Omega_{20} - m^2) + \tfrac{1}{2}b_1^2(\Omega_{02} - m^2) + N \quad \ldots \quad (10)$$

where N stands for this expression of the fourth order

$$\begin{aligned} N = {} & \tfrac{1}{2}\Omega_{20}(2a_0^2 + a_2^2) + \tfrac{1}{2}\Omega_{02}b_2^2 \\ & + \tfrac{1}{4}\Omega_{30}a_1^2(2a_0 + a_2) + \tfrac{1}{4}\Omega_{12}b_1(2a_0b_1 - a_2b_1 + 2a_1b_2) \\ & + \tfrac{1}{32}\Omega_{40}a_1^4 + \tfrac{1}{16}\Omega_{22}a_1^2b_1^2 + \tfrac{1}{32}\Omega_{04}b_1^4 - 2m^2(a_2^2 + b_2^2) \\ = {} & \tfrac{1}{4}(a_1L + b_1M) - 4m^2(a_2^2 + b_2^2) - 4ma_2b_2 \quad \ldots \quad (10') \end{aligned}$$

The eight equations (3), (4), . . . (10) are sufficient to determine m and the seven coefficients a_0, a_1, . . .

5. Since (3), (6), and (7) give a_0, a_2, and b_2 in terms of a_1, b_1, and m, the latter quantities are determined by (4), (5), and (10). An approximate solution is obtained by neglecting L, M, and N. This will give

$$\frac{a_1}{b_1} = \frac{-2m}{m^2 + \Omega_{20}} = \frac{m^2 + \Omega_{02}}{-2m} \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$0 = (m^2 + \Omega_{20})(m^2 + \Omega_{02}) - 4m^2 \quad \ldots \quad \ldots \quad (12)$$

$$a_1^2 = (m^2 + \Omega_{02})a^2 \; ; \; b_1^2 = (m^2 + \Omega_{20})a^2 \quad \ldots \quad (13)$$

$$C_0 - C = (m^4 - \Omega_{20}\Omega_{02})a^2 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

Then, if L, M, and N be reinstated, it will be necessary to replace a_1 by $a_1 + \delta a_1$, b_1 by $b_1 + \delta b_1$, and m by $m + \delta m$, the three increments being clearly of the second order. Hence

$$\begin{aligned} 0 &= (m^2 + \Omega_{20})\delta a_1 + 2m\delta b_1 + 2(ma_1 + b_1)\delta m + L \\ 0 &= 2m\delta a_1 + (m^2 + \Omega_{02})\delta b_1 + 2(mb_1 + a_1)\delta m + M \\ 0 &= (\Omega_{20} - m^2)a_1\delta a_1 + (\Omega_{02} - m^2)b_1\delta b_1 - m(a_1^2 + b_1^2)\delta m + N \end{aligned}$$

the quantities L, M, N and the coefficients in these equations being calculated with the approximate values of a_1, b_1, and m. Forms more convenient for calculation can be deduced by the help of (11), (12), and (13). Thus if

$$\frac{\delta a_1}{a_1} - \frac{\delta b_1}{b_1} = \delta_1, \quad \frac{\delta a_1}{a_1} + \frac{\delta b_1}{b_1} = \delta_2$$

the following form an equivalent set of equations :

$$(m^4-\Omega_{20}\Omega_{02})\delta m = m^2(b_1^{-1}\,\mathrm{L}+a_1^{-1}\,\mathrm{M}) \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

$$4m\delta_1 = (\Omega_{20}-\Omega_{02})\delta m + b_1^{-1}\,\mathrm{L} - a_1^{-1}\,\mathrm{M} \ldots \quad \ldots \quad (16)$$

$$(m^4-\Omega_{20}\Omega_{02})\delta_2 = m^2(\Omega_{20}-\Omega_{02})\delta_1 - m(2m^2+\Omega_{20}+\Omega_{02})\delta m + a^{-2}\,\mathrm{N} \quad \ldots \quad (17)$$

The first of these determines approximately the variation of the period from its limiting value ; the two latter give successively δ_1 and δ_2, whence δa_1 and δb_1 are obtained immediately. The coefficients a_3 and b_3 are found from (8) and (9), and thus the solution to the third order is complete.

6. To the derivatives of the force function given in the former paper must be added those of the fourth order. Their values are given by

$$3\Omega_{40} = -6\Omega_{22} = 8\Omega_{04} = 72(\mu r_1^{-5} + \nu r_2^{-5})$$

A table of numerical values for the case in which $11\mu = 10$, $11\nu = 1$, will now be given, many of the numbers being taken from the earlier paper :

	Family *a*.	Family *b*.	Family *c*.
r_1	0·71751	1·34700	0·94693
r_2	0·28249	0·34700	1·94693
C_0	3 65292	3·53418	3·17322
Ω_{20}	+ 13·9878	+ 6·0955	+ 3·1660
Ω_{02}	− 5·4939	− 1·5478	− 0·0830
$\Omega_{30} = -2\Omega_{12}$	+ 65·076	− 39·279	+ 6·822
$3\Omega_{40} = -6\Omega_{22} = 8\Omega_{04}$	+ 3982·86	+ 1315·79	+ 86·202
m	2·6082	1·6763	1·0706
$m4-\Omega_{20}\Omega_{02}$	123·13	17·330	1·5766
$a^{-1}\,.\,a_1$	− 1·1439	− 1·1234	− 1·0312
$a^{-1}\,.\,b_1$	+ 4·5597	+ 2·9841	+ 2·0766
$a^{-2}\,.\,a_0$	+ 10·5685	− 5·1400	+ 0·5887
$a^{-2}\,.\,a_2$	− 4·1352	+ 2·6261	− 0·5487
$a^{-2}\,.\,b_2$	− 1·9209	+ 1·5800	− 0·2895
$a^{-3}\,.\,\mathrm{L}$	+ 1234·6	+ 73·93	+ 2·8618
$a^{-3}\,.\,\mathrm{M}$	+ 3493·8	+ 47·45	+ 1·4737
$a^{-4}\,.\,\mathrm{N}$	+ 2981·3	− 118·74	− 2·4169
$a^{-3}\,.\,A_3$	− 2044·8	− 288·49	− 8·3943
$a^{-3}\,.\,B_3$	− 2190·5	− 225·81	− 6·5525
$a^{-2}\,.\,\delta m$	− 153·79	− 2·832	− 0·03708
$a^{-3}\,.\,\delta a_1$	− 92·47	− 6·227	− 0·3670
$a^{-3}\,.\,\delta b_1$	+ 224·79	− 3·652	− 0·5638
$a^{-3}\,.\,a_3$	+ 20·19	+ 7·109	+ 0·4530
$a^{-3}\,.\,b_3$	+ 33·64	+ 6·500	+ 0·3559

7. In the former paper the results for orbits of family a were compared with four orbits given by Professor Darwin. The comparison can now be repeated with improved figures. The values of the energy constant C and the parameter a are given by

$$\begin{aligned} 11\,C &= 40{\cdot}0, \quad 39{\cdot}0, \quad 38{\cdot}5, \quad 38{\cdot}0 \\ \log a &= 8{\cdot}06428,\ 8{\cdot}47046,\ 8{\cdot}54705,\ 8{\cdot}60356 \end{aligned}$$

The general equations for the family are

$$\xi = +10{\cdot}5685a^2-(1{\cdot}1439a+92{\cdot}47a^3)\cos mt-4{\cdot}1352a^2\cos 2mt +20{\cdot}19a^3\cos 3mt$$

$$\eta = (4{\cdot}5597a+224{\cdot}79a^3)\sin mt-1{\cdot}9209a^2\sin 2mt +33{\cdot}64a^3\sin 3mt$$

$$m = 2{\cdot}6082-153{\cdot}79a^2$$

which become in the four cases considered

(α) $\xi = +{\cdot}0014-{\cdot}0134\cos mt-{\cdot}0006\cos 2mt+{\cdot}0000\cos 3mt$
$\eta = +{\cdot}0533\sin mt-{\cdot}0003\sin 2mt+{\cdot}0001\sin 3mt$
$m = 2{\cdot}588$

(β) $\xi = +{\cdot}0092-{\cdot}0362\cos mt-{\cdot}0036\cos 2mt+{\cdot}0005\cos 3mt$
$\eta = +{\cdot}1405\sin mt-{\cdot}0017\sin 2mt+{\cdot}0009\sin 3mt$
$m = 2{\cdot}474$

(γ) $\xi = +{\cdot}0131-{\cdot}0443\cos mt-{\cdot}0051\cos 2mt+{\cdot}0009\cos 3mt$
$\eta = +{\cdot}1705\sin mt-{\cdot}0024\sin 2mt+{\cdot}0015\sin 3mt$
$m = 2{\cdot}418$

(δ) $\xi = +{\cdot}0170-{\cdot}0519\cos mt-{\cdot}0067\cos 2mt+{\cdot}0013\cos 3mt$
$\eta = +{\cdot}1975\sin mt-{\cdot}0031\sin 2mt+{\cdot}0022\sin 3mt$
$m = 2{\cdot}361$

The following table gives the points where the orbits cross the axis of x (ξ_0 and ξ_1), the points (ξ', η') for which $mt = \frac{1}{2}\pi$ and the angle $(2\pi m^{-1})$ through which the system turns in one period of the satellite. In the comparison numbers η' is the ordinate corresponding to the calculated ξ'.

$mt=$	0	π	$\frac{1}{2}\pi$		
	ξ_0	ξ_1	ξ'	η'	$2\pi m^{-1}$
(α)	$-{\cdot}0126$	$+{\cdot}0142$	$+{\cdot}0020$	$+{\cdot}0532$	$139^\circ 1$
D.	$-{\cdot}0125$	$+{\cdot}0147$	,,	$+{\cdot}0530$	$138^\circ 3$
(β)	$-{\cdot}0301$	$+{\cdot}0413$	$+{\cdot}0128$	$+{\cdot}1396$	$145^\circ 5$
D.	$-{\cdot}0304$	$+{\cdot}0413$	,,	$+{\cdot}1395$	$145^\circ 7$

$mt=$	0	π	$\frac{1}{2}\pi$		
	ξ_0	ξ_1	ξ'	η'	$2\pi m^{-1}$
(γ)	−·0354	+·0514	+·0182	+·1690	148°·9
D.	−·0361	+·0523	„	+·1691	149·6
(δ)	−·0403	+·0609	+·0237	+·1953	152·5
D.	−·041	+·060	„	+·197	(not determined)

The limiting value of $2\pi m^{-1}$ for an infinitesimal orbit is 138°·0. The agreement shown in the above table is much better than in the corresponding table of the previous paper (vol. lxiii. p. 442), and may be considered satisfactory.

8. For the two orbits of family *b* given by Professor Darwin the values of C and a are given by

$$\begin{aligned} 11\,C &= 38{\cdot}5, \qquad 38{\cdot}0 \\ \log a &= \ 8{\cdot}64750, \quad 8{\cdot}83116 \end{aligned}$$

The general equations for the family are :

$$\xi = -5{\cdot}1400\,a^2 - (1{\cdot}1234a + 6{\cdot}227a^3)\cos mt + 2{\cdot}6261a^2\cos 2mt + 7{\cdot}109a^3\cos 3mt$$

$$\eta = (2{\cdot}9841a - 3{\cdot}652a^3)\sin mt + 1{\cdot}5800a^2\sin 2mt + 6{\cdot}500a^3\sin 3mt$$

$$m = 1{\cdot}6763 - 2{\cdot}832a^2$$

which become in the two cases considered

(α) $\xi = -{\cdot}0101 - {\cdot}0504\cos mt + {\cdot}0052\cos 2mt + {\cdot}0006\cos 3mt$

$\eta = +{\cdot}1322\sin mt + {\cdot}0031\sin 2mt + {\cdot}0006\sin 3mt$

$m = 1{\cdot}6707$

(β) $\xi = -{\cdot}0236 - {\cdot}0781\cos mt + {\cdot}0121\cos 2mt + {\cdot}0022\cos 3mt$

$\eta = +{\cdot}2012\sin mt + {\cdot}0073\sin 2mt + {\cdot}0020\sin 3mt$

$m = 1{\cdot}6633$

The comparison is made as in the preceding paragraph

$mt=$	0		$\frac{1}{2}\pi$		
	ξ_0	ξ_1	ξ'	η'	$2\pi m^{-1}$
(α)	−·0547	+·0449	−·0153	+·1316	215°·5
D.	−·0551	+·0445	„	+·1312	[213·9]
(β)	−·0874	+·0644	−·0357	+·1991	216·4
D.	−·0875	+·0679		+·1987	208

The limiting value of $2\pi m^{-1}$ for an infinitesimal orbit is 214°·8. There is therefore a discrepancy between the two methods i

regard to the periods of the orbits, Professor Darwin's results indicating a decrease of the period as the orbits increase in size. This is not supported by my figures. The value [213°·9] really belongs to a neighbouring orbit which is not quite periodic, so that this number is not strictly comparable. On the other hand the orbit (β) contains in the value of ξ_1 a second discrepancy, which is greater than the evidence based on other comparisons would lead us to expect. On the whole it seems quite possible that the determination of this orbit by the method of quadratures has not been so accurate as in other cases.

9. In Professor Darwin's results the range of the energy constant has the limits given by $11C = 40{\cdot}0$ and $11C = 38{\cdot}0$. Now orbits of family *c* only begin to appear after $11C = 35{\cdot}0$, and therefore orbits of this family for comparison are wanting. But, although the results cannot be submitted to this kind of check, it seems worth while to apply them to drawing a few new orbits of this family. The general equations are

$$\xi = +{\cdot}5887\,a^2 - (1{\cdot}0312\,a + {\cdot}3670\,a^3)\cos mt - {\cdot}5487\,a^2\cos 2mt + {\cdot}4530\,a^3\cos 3mt$$

$$\eta = (2{\cdot}0766\,a - {\cdot}5638\,a^3)\sin mt - {\cdot}2895\,a^2\sin 2mt + {\cdot}3559\,a^3\sin 3mt$$

$$m = 1{\cdot}0706 - {\cdot}03708\,a^2.$$

The selected values of C and a are given by

$11C =$	34·5	34·0	33·5	33·0
$\log a =$	9·18439	9·35886	9·45434	9·52043

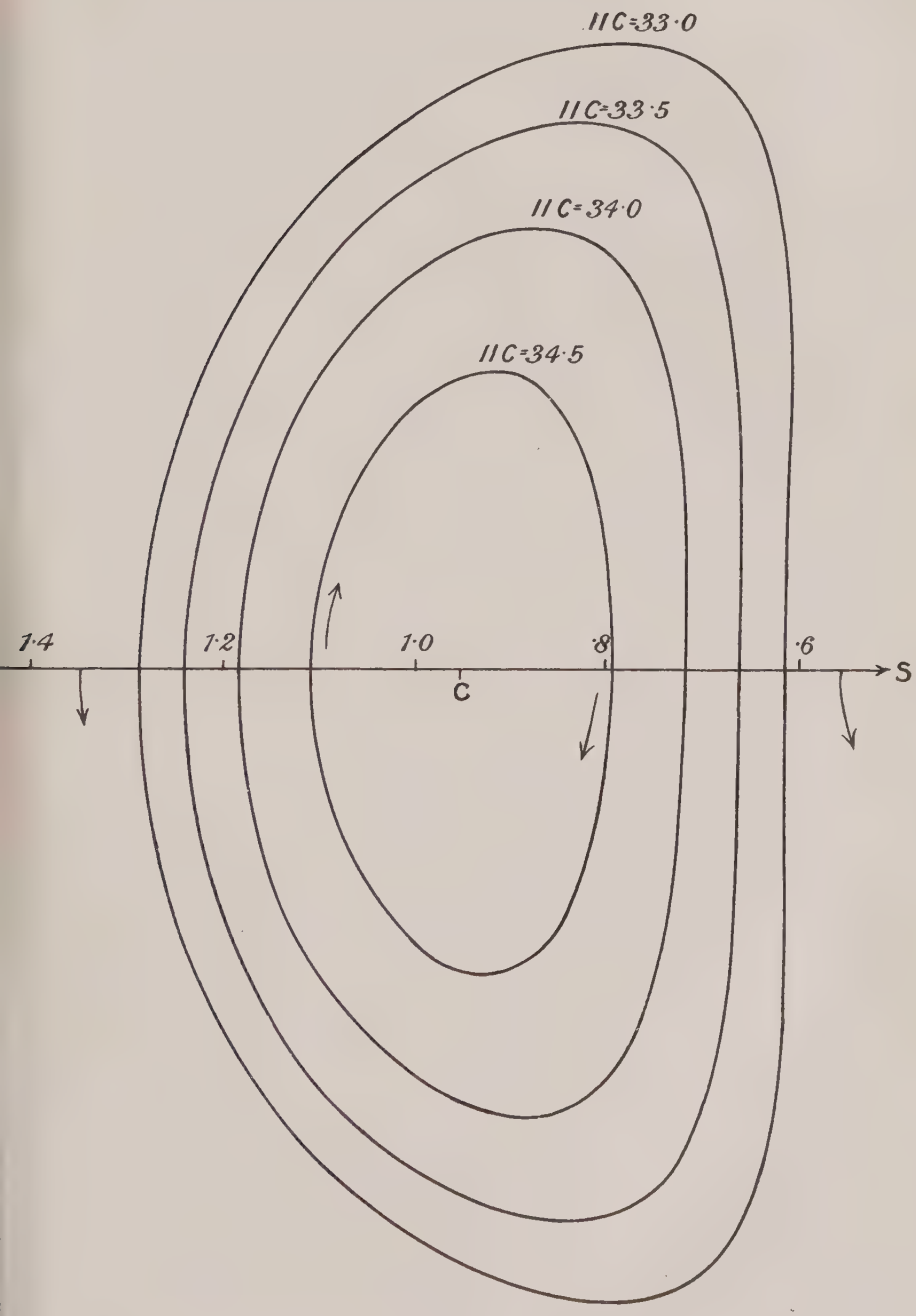
The equations of the four orbits then become

(α) $\xi = +{\cdot}0138 - {\cdot}1590\cos mt - {\cdot}0128\cos 2mt + {\cdot}0016\cos 3mt$
$\eta = +{\cdot}3155\sin mt - {\cdot}0068\sin 2mt + {\cdot}0013\sin 3mt$
$m = 1{\cdot}0697\,;\ 2\pi m^{-1} = 336°{\cdot}55$

(β) $\xi = +{\cdot}0307 - {\cdot}2400\cos mt - {\cdot}0286\cos 2mt + {\cdot}0054\cos 3mt$
$\eta = +{\cdot}4678\sin mt - {\cdot}0151\sin 2mt + {\cdot}0042\sin 3mt$
$m = 1{\cdot}0687\,;\ 2\pi m^{-1} = 336°{\cdot}85$

(γ) $\xi = +{\cdot}0477 - {\cdot}3020\cos mt - {\cdot}0445\cos 2mt + {\cdot}0105\cos 3mt$
$\eta = +{\cdot}5782\sin mt - {\cdot}0235\sin 2mt + {\cdot}0082\sin 3mt$
$m = 1{\cdot}0676\,;\ 2\pi m^{-1} = 337°{\cdot}20$

(δ) $\xi = +{\cdot}0647 - {\cdot}3552\cos mt - {\cdot}0603\cos 2mt + {\cdot}0165\cos 3mt$
$\eta = +{\cdot}6678\sin mt - {\cdot}0318\sin 2mt + {\cdot}0130\sin 3mt$
$m = 1{\cdot}0665\,;\ 2\pi m^{-1} = 337°{\cdot}55$


IIC=33·0
IIC=33·5
IIC=34·0
IIC=34·5
1·4
1·2
1·0
·8
·6
C
S

The curves corresponding to these equations are represented in the accompanying plate (Plate 2), in which the unit of length is 5 in., this being the scale on which most of Professor Darwin's orbits are reproduced.

10. A comparison of §§ 7 and 8 with §§ 10 and 11 of the previous paper will show the degree of improvement obtained for families *a* and *b* by increasing the order of approximation, and give some idea of the convergence (in a practical and not technical sense) of the series. It has now been found possible to calculate the variation of the period, an element which, with the lower approximation, is supposed constant for each family. In every case the period is found to increase with the scale of the orbits, the variation being most marked in the case of family *a* and least in the case of family *c*. As regards the shape and position of the orbits, the general features which characterise the departure from the small elliptic paths surrounding the centres of libration are similar in all cases. The mean position of the satellite tends always in the direction of the nearer body of finite mass, while at the intersection of the orbit with the line of syzygies in the same direction, the curvature first decreases and finally changes sign. These features are shown, though not in a very marked degree, by the orbits of family *c* drawn in this paper. Where comparison was possible with the results of Professor Darwin's researches it has been made, and in one case among the six orbits available a discrepancy is found which is pointed out in § 8. The method of the present paper is extremely simple and elementary ; yet it might be extended without serious difficulty to higher orders of approximation if this were desirable. The next order in particular might easily be added. But it has not seemed likely that anything of value would be gained thereby. A series of orbits of oscillating satellites has been found by M. Burrau (*A.N.* 3251) to be limited by a "trajectory of ejection." The real obstacle in the way of following up to this limit the continuation of the class of solutions considered in this paper is the limited range over which the force function can be expanded in ascending powers of the relative coordinates. Whether this difficulty can be surmounted is a question which must be left for further consideration.

University Observatory, Oxford:
1903 *December* 7.

Preliminary Note on the Effect of the Direction of Gravity on Lunar Observations. By E. B. H. Wade, M.A.

(*Communicated by Professor H. H. Turner.*)

That the Moon's apparent place is affected by the spheroidal figure of the Earth is well known. If $d\phi$ is the difference between geocentric and geographic latitude, the Moon's declination on the meridian requires a supplementary correction of $p \times \cos\zeta\ d\phi$, which in extreme cases amounts to $1''$ for every $1'$ of $d\phi$.

Unless this correction is made, latitudes found by altitudes of the Moon will differ to that extent from those found by altitudes of stars. Again, if there be admitted a distinction between geocentric and geographic longitude ($d\lambda$), then the Moon's R.A. at transit requires a correction for parallax $p\ \cos t\ \frac{\cos\phi}{\cos\delta}\ d\lambda$, instead of being free from parallax as usually assumed. For every $1'$ of $d\lambda$ this means, in equatorial latitudes, $1''$ error about in R.A. Now since longitudes found by Moon culmination are about thirty times as erroneous as the R.A. from which they are deduced, we find that if $d\lambda$ attains $1'$, the longitude will differ 2^s from that given by star-signals. If this has not been emphasised before, it is no doubt due to the practical certainty that $d\lambda$ cannot attain to $1'$, but we shall nevertheless pursue the idea somewhat further.

The 2^s in longitude, though a large quantity, means only $1''$ in R.A., not a large quantity perhaps to detect by observation with the transit instrument. In the case of occultations near the meridian, the disappearance would be accelerated or delayed by 2^s, which could not escape observation, as times of disappearance are reliable to $0^s{\cdot}2$. We conclude that a difference of only $6''$ between geocentric and geographical longitude has a measurable effect on an occultation.

Application.

It is immaterial to the above argument whether the $d\phi$ or $d\lambda$ arise from (*a*) systematic ellipticity, (*b*) accidental causes (so-called local attraction), (*c*) a regional drift of gravity prevailing over large areas, or any combination of these.

(*a*) We have shown that if this effect attain to $6''$ in longitude it will be noticeable, but it would only be detected in geodetic operations if correct to 1 in 45,000, both in the triangulation and in the astronomical work.

(*b*) Is sufficiently detected by ordinary geodetic operations.

(*c*) Might suitably be dealt with by lunar observations.

Major Burrard, R.E., in a paper read to the British Association (*B.A. Report*, 1902, p. 541), expresses a fear that a large

regional drift may remain uneliminated, so that it is not possible to apply a map of India to a map of the whole world with certainty. As this doubt has been authoritatively expressed, it seems worth while to point out that lunar observation would detect such drift, whether in latitude or longitude. In the case of latitude, that found from the Moon would differ by 1″ from that found from stars, and in the case of longitude that found from occultations would differ 2^s (30″) from that found from star signals, if $d\phi$ or $d\lambda$ attain to 1′.

Or, better, on the same night, longitudes found from occultations at large hour-angles would differ nearly 2^s (30″) from those observed near the meridian.

Occultations appear to be very suitable, but as long as they are corrected for ephemeris errors obtained with transit instruments, they are no better than transit observations. In the case when two occultations are observed on one night, or one in two localities, this objection is removed.

It may be added that a marked discrepancy was found in the fundamental Indian longitude based on a long series of Moon culminations, and the same later determined telegraphically, and this note may help to explain it.

Abbasia Observatory, Egypt: 1903 *October* 26.

Remarks on a Paper by Mr. Cooke on a New Method of Determining Time, Latitude, and Azimuth. By E. B. H. Wade, M.A.

(*Communicated by Professor H. H. Turner.*)

At the beginning of the present year Mr. Cooke described a new method of determining time, latitude, and azimuth (*Monthly Notices, R.A.S.*, vol. lxiii. p. 156). As I have been employing precisely his method since May 1902, I thought it would be permissible to make a few additional remarks on the subject.

Having been instructed to determine the latitude of Tamia in the Fayum Province of Egypt, I proceeded thither with an 8-inch theodolite, intending to rely on circle readings. However, a very strong wind was blowing, and this caused such inconvenience in illuminating graduations, that Talcott's method seemed preferable. For this a vertical micrometer is necessary, and the one furnished with my instrument could not be turned into the vertical without structural alterations. I therefore decided to make my chronometer a substitute for the micrometer. For example, if the second of a pair of stars pass the meridian at a greater altitude than the first, then by placing the instrument out of the meridian for the second star we may still secure it at the same altitude as the first. It at once became evident that

pairs of stars could thus be utilised which would not come within the limited range of Talcott's method. Moreover, it was now necessary to know the error of the chronometer, and for this purpose I included stars in the neighbourhood of the prime vertical. Thus, by degrees, I had come to employ as a substitute for Talcott's method, one differing in no respect from that described by Mr. Cooke.

As the results were satisfactory, I contemplated publishing them, but a study of the literature of the subject led me to think that there was little if anything original in the method. I therefore contented myself with writing a departmental minute on the subject, and inserting an appendix on it in the annual report of the Observatory for 1902. In the first place the method differed only from Chandler's* in the use of a spirit level ; again, Chauvenet had described a method for finding latitude and time by the observation of three or more stars at equal altitudes.† At the time when he wrote, Chandler's device had not been invented, and the instrument which he had in mind must have been a sextant, a zenith telescope, or a theodolite. Even if he had not a theodolite in mind, the use of a theodolite as zenith telescope did not appear to me new. For instance, Sir David Gill has used theodolites in place of zenith telescopes on the survey of South Africa.‡ However, the method, even if not strictly new, has probably escaped the notice of surveyors, and it is therefore fortunate that Mr. Cooke has drawn attention to the subject.

Applications.

In my own case I have used the method for latitudes and time only. I deliberately rejected it as a means of obtaining azimuths.

Mr. Cooke refers to the difficulties arising from circle graduations and collimation. To these we may add the reading of a striding level and the pivot errors. All these must be reckoned with to obtain the azimuth of a terrestrial mark. I have preferred a method specially designed to mitigate these evils. An account of it will appear in the report above referred to.

Definitions.

In this note I propose to use the following definitions :—

ϕ = observer's latitude.

θ = sidereal time of observation.

h = star's altitude, A = star's azimuth } at time θ.

δ = star's declination.

* *Annals of Harvard College Observatory*, vol. xvii.

† *Spherical and Practical Astronomy*, vol. i. p. 280.

‡ *Report on the Geodetic Survey of South Africa*, 1894.

Practice.

My practice appears to be the same as Mr. Cooke's in all details except the following :—

1. I usually point the instrument to *Polaris* at the beginning of the set, instead of the altitude ϕ + refraction. But the method of calculation (given below) leaves me quite free in this matter, and the set may be taken at any convenient altitude. If the method is to apply to equatorial latitudes, the constant altitude must evidently differ much from that of the pole.

2. The azimuth circle is never read for the reasons stated above. I would add that if it is to be read, and the observer's object is to find latitude and azimuth only and in one operation, the timekeeper may be dispensed with, which might be a great convenience in the field. Consider a special case. A star of known declination north of the zenith cuts the circle of equal altitude twice, and the horizontal readings are A_1A_2. Another to the south does so at readings A_3A_4. Either $\frac{A_1+A_2}{2}$ or $\frac{A_3+A_4}{2}$ is the meridional reading of the theodolite, and the relation between A_1-A_2 and A_3-A_4 leads to the latitude. A general theory for any number of stars is easily worked out.

Computation.

From the fact that we have been working independently, my method of computation is totally different from Mr. Cooke's. I begin by assuming a probable value for the latitude of the place and the error of my chronometer. I then calculate on these assumptions the zenith distance of every star in the set from the formula,

$$\sin h = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t.$$

I make no attempt, as does Mr. Cooke, to group my stars into pairs. All stars are calculated in exactly the same way. Now, supposing that the assumption as to time and latitude is correct, and further that the "bubble" readings are throughout the same, and that the error of observation is negligible, then all these computed zenith distances will work out exactly equal ; but, if not, each star will give us an equation of the form,

$$h + \Phi d\phi + \mathrm{T}dt - h_o = 0.$$

In these equations the quantities Φ and T are the differential coefficients expressing the result of errors of chronometer or latitude upon the calculated zenith distance. These coefficients may be obtained by the ordinary rules of spherical trigonometry, and the unknown quantities, $d\phi$, dt, h_o, may then be found by the method of least squares. In my earliest observations I

proceeded thus, but in later ones I have employed the modification about to be described.

In the table below the first column is the ordinary calculation of an altitude. The second shows the changes (in units of the seventh place) produced by increasing ϕ by 10″ ; these are found at once from the proportional parts 44 and −10 for sin ϕ and cos ϕ respectively. Similarly, the third column shows the effect of an increase in t by 10″ ; it is found from the proportional parts for cos t ; namely, 104.

	I.	II.	III.
log sin ϕ	1. 6377 167	44	...
log sin δ	1. 8075 742	...	...
sum	1. 4452 909	44	...
nat (1)	0. 2757 991	29	...
log cos ϕ	1. 9546 301	−10	...
log cos δ	1. 8846 006	...	...
log cos t	1. 2958 659	...	−104
sum	0. 1350 966	−10	−104
nat (2)	0. 1364 887	− 3	− 33
(1)+(2)	0. 4152 878	26	− 33
log (1+2)	0. 6153 492	27	− 35
h =	24° 32′ 14″, 8	6″, 0	−7″, 5

The equation for the star is therefore

$$0{\cdot}60\, d\phi - 0{\cdot}75\, dt + 14''{\cdot}8 - h_o = 0$$

Results.

I proceed to give some results with the method. I would first mention that they are, strictly speaking, field results. Without wishing to be unduly critical, I question whether Mr. Cooke's results were obtained at such a distance from his observatory that they can really be compared with field results obtained by other observers. Working at hundreds of miles from the nearest centre of civilisation, we are beset with petty difficulties which tend to diminish accuracy. The further, however, that we go into the field, the more we are struck by the suitability of the present method for field purposes.

1. *Station Tamia, June* 21, 1902.

Latitude found 29° 28′ 0″ − 4″, 65. $\Delta\theta = +46''\ 2.$

Residuals given by stars

	″
β Cassiopeiæ	2·1
α Cassiopeiæ	2·4

ξ Capricorni	3·6	
γ Capricorni	1·0	East of meridian.
β Capricorni	−1·7	
γ Capricorni	−4·3	West of meridian.
δ Cassiopeiæ	−2·2	
ι Herculis	−2·2	

This example is given chiefly on account of its early date.

2. *Station near Luxor.*

	Latitude found.	$\Delta\theta$.
	° ′ ″	″
1903 Feb. 26	25 44 12, 0	+11, 0
27	10, 4	+0, 0
28	11, 0	+12, 0

In this set as the result of increased experience the residuals are very much smaller than at Tamia. On the last two nights no single residual exceeded 1″·5.

The observations near Luxor are more suitable than those at Tamia for a discussion of accuracy. In the first place, there are three nights of observation; and secondly, the result can be compared with an independent method. Owing to the clearness of the sky in Egypt it was found possible to observe the altitude of the pole star in broad daylight, reading the circles of the instrument with micrometer microscopes under the most favourable conditions as to illumination. The arc of the instrument employed in this set was subsequently examined at the Observatory, and the following latitudes were obtained by the ordinary methods. For convenience the results obtained by the two methods are placed side by side :

	By Polaris.	By Method.		By Polaris.	By Method.
	° ′ ″	″		° ′ ″	″
Feb. 25	25 44 10·7		Feb. 26	25 44 11·4	
	12·2			10·8	
	11·0	...			
	11·0		27	12·0	
				11·3	10·4
26	9·9			12·5	
	10·8				
	8·4	12·0	28	—	11·0
	9·8				

Mean by Polaris.	Mean by Method.
° ′ ″	° ′ ″
25 44 10·9	25 44 11·1

Apparently the accuracy which I have reached is rather less than Mr. Cooke's, but my results, I think, confirm his opinion as to the convenience of the method. I may add that the times which I have obtained by this method have only been needed for approximate purposes, but I believe them to be quite accurate. In May of the present year I received instructions to determine the latitude and longitude of Tema, and made my arrangements to obtain both latitude and accurate local time by the method; but the telegraphic arrangements fell through, so that the method was only applied to the determination of latitude. I am convinced, however, that more accurate local times can be obtained in this way than by any other field method.

Relative Accuracy of Mr. Cooke's and other Methods.

It is interesting to inquire to what circumstance this method owes its supposed superiority over other field methods, including Talcott's. Given a faultless micrometer, Talcott's is a special case of Mr. Cooke's, and the one in which the error in zenith distance has the minimum effect on the latitude. If the superiority is confirmed, it would seem to imply that the accuracy of Talcott's method has hitherto been limited by micrometer errors.

My best thanks are due to Professor Turner for encouragement in this investigation, to the Survey Department of Egypt for providing facilities for the observations, and to the Under Secretary of State for Egyptian Public Works for permission to publish this note.

Abbasia Observatory, Egypt:
1903 *October.*

On the use of the Stereo-comparator for Plates on which a Réseau has been Impressed. By Dr. Max Wolf.

[*Introductory Note by Professor H. H. Turner.*—The accounts given of the ease and rapidity with which two plates of the same region can be compared with the stereo-comparator naturally create a desire to possess such an instrument. A generous offer has been made to present one to the Oxford University Observatory, *provided that it will clearly be of use in our work.* The very proper condition specified at once raises the question whether plates which have a réseau impressed upon them, and which have several images of the same star, can be scanned as readily by the stereoscopic method as those containing only a single image of each star and no réseau; for all our plates accumulated hitherto at Oxford are of the former class. For some time I hoped to be able to obtain a stereo-comparator on loan to make

the experiment ; the makers (Messrs. Zeiss) courteously offered to put at my disposal for this purpose the first one available. But the demand for the instruments has been, I am glad to learn, so great, that hitherto they have been unable to do more than keep pace with it ; and ultimately I ventured to ask Dr. Max Wolf to make the experiments for me, which he very kindly consented to do. Two plates were sent him, both impressed with a réseau, and containing, the first three, and the second two, images of each star. They had been exposed for the planet *Eros*, and thus afforded material for tests of various kinds. It will be seen from the following letter that, even in this extreme case, the stereo-comparator can be used with advantage under certain conditions. The letter suggested that if *one* of the plates were free from a réseau, many of the difficulties would disappear, and this was found to be the case. A second pair of plates was sent to Heidelberg, one with a réseau, the other without, and a mere glance was sufficient to show Dr. Wolf that "there was now no trouble in observing two such plates with the stereo-comparator. The best position is to bring the chains of three stars in the line joining the eyes, and then it gives a splendid effect, so that the examination of the plates is easily made." In proof of which, Dr. Wolf remarked that there was a planet on one of the plates, and indicated its approximate position. The place was carefully measured when the plates were returned to Oxford, and the results—which seem to show that the planet was *Ausonia* (63)—have been communicated to the *Astronomische Nachrichten*. Hence, although it will be seen below that Dr. Max Wolf expresses himself rather doubtfully as to the value of the stereo-comparator for cases when both plates have a réseau, these doubts need not seriously concern us. For the instrument can be brought into effective use by taking a new plate without a réseau.

But Dr. Max Wolf's account of his experiments when both plates have a réseau will probably interest others ; and I therefore give it at full length.

I should add that Dr. Pulfrich of Jena also offered to make similar experiments, and another pair of plates was accordingly sent to him. His full account of the examination substantially agrees with Dr. Wolf's results. The réseau *disturbs* the examination, but does not *prevent* it—this is the verdict of both.

As a result of these experiments I felt justified in assuring Mr. C. L. Brook, F.R.A.S., who had made the generous offer above referred to, that the instrument would be most useful to us ; and he promptly purchased it for the Oxford University Observatory. It was on view at the R.A.S. rooms on November 27 and following days.]

Dear Professor Turner,—It is indeed a very difficult problem which you have set to me, and I can hardly give you advice whether to procure a stereo-comparator or not. Indeed,

a réseau makes the use of a stereo-comparator much more difficult than it is for our plates without réseau.

You have forwarded me two plates No. 1619 and 1620. No. 1619 contains of each star three images, No. 1620 two images :

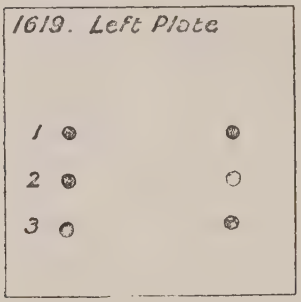


Fig. 1.

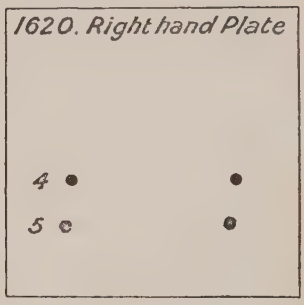


Fig. 2.

Now, if I put the two plates as seen here in the stereo-comparator and adjust the plates so that the middle images of 1619 coincide with the upper images of 1620 (viz. 2 with 4), then the star 1 comes out of the "infinity" plane and is close to the observer,* but the point 3 (combined with 5) approaches also to the observer, but not so much as the point 1, because the distance 2–3 is not equal to the distance 4–5.

So it happens that you see (of each star) always three images freely suspended in space : No. 1 very near to the observer, No. 2 at a very great distance far behind, and No. 3 at a mean distance between the two.

This appearance is preserved all over the plate if you move the two slides of the stereo-comparator ; you always see this beautiful roomy chain ; and at any place, where there is some trouble, you see it immediately by the change of this order.

On the two plates is also the planet *Eros*. Like all the stars he has made on 1619 three images, on 1620 two. Since some time has elapsed between the two exposures, the pictures do not coincide, but are situated at wholly different places. On each plate, therefore, the planet is seen immediately by the stereo-effect. On the plate with the three images *all three* points of *Eros* come far out of the ∞ plane and seem to be as near to the observer as points No. 1 of the stars. The same is with the left plate (I did not know the positions before).

In the same manner every wrong star is easily detected by the stereo-comparator. The stereo-effect is, of course, not necessary to detect faint stars, because all stars are threefold,

* There is, of course, no direct stereoscopic effect for an object which appears on one plate only ; but the appearance suggested is that of an object close to the observer.

and every spot only once upon the plate. But, notwithstanding, it is very comfortable to go over the plate using the slides of the stereo-comparator, and to feel the errors without thinking.

I am sorry that I could not detect a variable star on the plates, but there seems to be no variable.

Going over the plate I found the following wrong stars, which are all caused by holes in the réseau plate:

$x=15{\cdot}5$	:	$y=2{\cdot}4$
$25{\cdot}8$		$10{\cdot}8$
$0{\cdot}8$		$10{\cdot}5$
$20{\cdot}4$		$11{\cdot}2$
$21{\cdot}6$		$13{\cdot}7$=(artificial cluster)
$20{\cdot}2$		$17{\cdot}0$

&c.

These spots are on the two plates at the same places of the réseau, so that it is certain that they are holes in this.

So far as this, the stereo-comparator is of great value to find out changes in the stars and errors of the plates.

But now we come to the difficulty!

If the stereo-comparator is adjusted to the stars as above, the lines of the two réseaux fall tolerably separate; and after practice for a day or so, one is accustomed to neglect the lines and to see the stars free in space on a distant indefinite network. If you look firmly upon the net then, you see the following view:

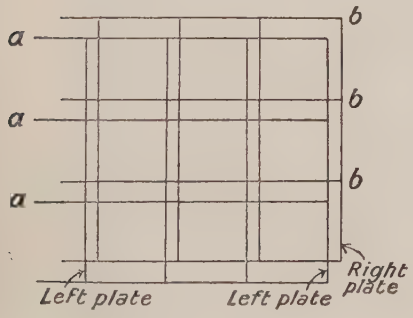


Fig. 3.

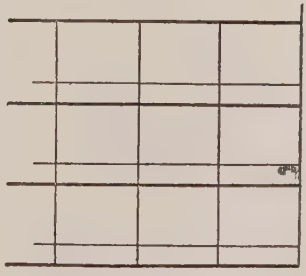


Fig. 4.

Fig. 3 gives the relative situation roughly of the two nets of the two plates, if the stars are adjusted as described above; Fig. 4 gives the view seen with the two eyes by the stereo-comparator. We see the vertical lines join together, whilst the horizontal remain separated. This results from the situation of the two eyes, which are in a horizontal. The horizontal lines of

the réseau of the left plate seem much thicker and somewhat nearer to the observer.

If starpoints 2 and 4 coincide over the whole plate, and if then the réseaux are adjusted absolutely parallel in the two

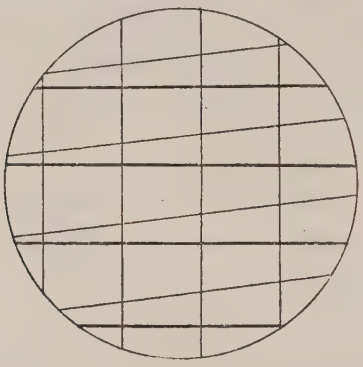


Fig. 5.

pictures, then we see in the stereo-comparator the horizontal lines not parallel to the eyes. It looks as if the lines *b* were inclined to the lines *a* (fig. 3). This is a very striking appearance. (Fig. 5 somewhat exaggerates the appearance.) The reason was found very soon. The connecting lines 1, 2, 3 and 4, 5 (figs. 1 and 2) are not parallel, and not parallel to the réseau. You see how the stereo-comparator can give the *errors of the orientation of your apparatus.*

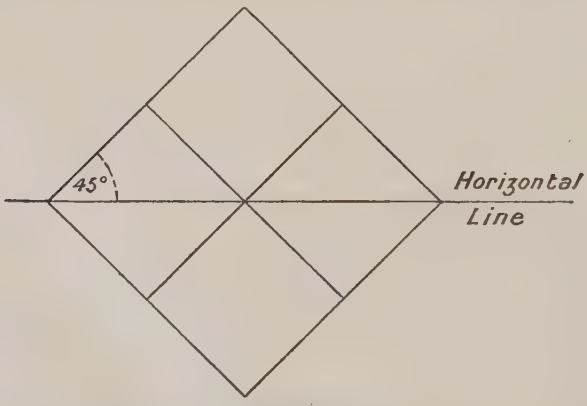


Fig. 6.

If we proceed to another adjustment of your plates we have still further difficulties. We will now put the two plates in the stereo-comparator, so that the lines of the réseau are inclined 45° to the connecting-line of your eyes (horizontal), fig. 6. If we

now adjust the plates so that the starpoints 2 and 4 coincide all over the plate, then all the lines join together in our mind, and we see only *one* net, *one* réseau ; but we cannot any longer see the stars coincident at the same time. By some effort it is possible to look firmly upon the stars, and to receive again the stereo-view of the stars. This is done most easily by moving the slides of the stereo-comparator and regarding the stars.

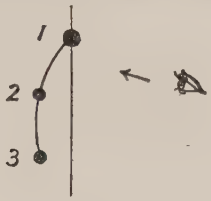


Fig. 7.

Point 1 is again very near to the observer.
2 very far distant.
3 lies nearer than at the first adjustment.

Eros seems very near to the observer. Pictures 3 and 5 of *Eros* seem very much nearer than the others.

The third position of the stereo-comparator was the following,

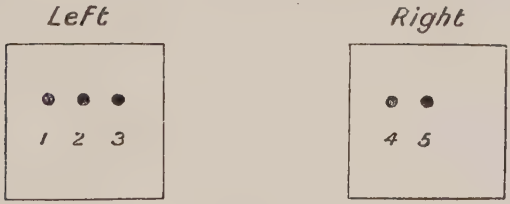


Fig. 8.

adjusting 2 upon 4 all over the plate. In this position picture 1 was still nearer to the observer, picture 2 still further, the effect a maximum, as natural. But now it was nearly impossible to seek over the plate and to fix the eyes upon the stars. At every moment the lines of the réseau will coincide in the eyes, and it is impossible for the eyes to hold upon the stars and to join the images of the stars in the mind.

Now you see the difficulty. If the two plates are so exposed that the lines fall *near* together when the stars are in coincidence, then the eyes are unable to hold the lines from together and to join the stars. If, on the other hand, the plates are so exposed that the lines are *far* from together, regarding the same starpoints, then the stereo-comparator is *very* useful and easily applied for the research of the plates.

You see also from the above that for these two plates, 1619 and 1620, there was one position which was useful; and it is not impossible that on a great part of your plates you will be able to find one position for stereo-comparing work.

Now you can imagine how difficult it is for me to bear the responsibility to give you advice whether to procure a stereo-comparator or not. There is no doubt you will not have the full benefit of the instrument, because in many cases the réseau makes the use of the stereo-comparator impossible; on the other hand, in many cases you will have some great *profit* of its use.

Excuse me that I can give you no deciding answer, but the reason lies not in my hands, it reposes on the difficulty of the matter.

With kind regards, yours very truly,

MAX WOLF.

Heidelberg: 1903 *July* 5.

An Examination of the Relative Star-density in Different Parts of the Plates forming the Harvard Photographic Sky-map. By J. C. W. Herschel, B.A.

In *Monthly Notices*, lxii. 1902 April, p. 444, Professor Turner gave diagrams for five different observatories, which use essentially similar objectives, showing that the relative number of star images photographed in different parts of a plate varies by about 50 per cent., and can be expressed in terms of the distance from the centre; there is no continuous area of useful uniformity; on the other hand, he showed that the *doublet* used for the C.P.D. gives a sensibly uniform field over $6° \times 6°$.

Professor Pickering has recently issued a set of fifty-five photographs taken with a doublet as "a photographic map of the entire sky" (*Harvard Circular*, No. 71), being picked plates chosen out of those with which a continuous watch has been kept upon the sky at Harvard College and Arequipa for many years.

It seems useful, therefore, to ascertain whether any considerable area of *these* plates has reasonable uniformity of star-density, and if so to what extent.

I am indebted to Professor Turner for suggesting this question, and for the use of the set of plates at the Oxford University Observatory.

The photographs were taken with a portrait lens of one inch aperture and thirteen inches focal length, and they cover about 30° square. Twelve plates (Nos. 1, 2, 6, 10, 16, 22, 28, 34, 40, 46, 50, and 54) were chosen, to cover an entire belt of the sky passing through the poles and cutting the Milky Way nearly at right angles; in fact, the plate centres lie along the equinoctial colure, at declinations $+60$, $+30$, 0, -30, -60, on both the 0^h and xii^h

circles, and one at $0^h+90°$ and $9^h-85°$. The plates not having réseaux themselves, a separate plate with a 5 mm. réseau of thirty-two squares to a side was used, and this covered most of the photograph. Some difficulty was found at first owing to grit in the film of the réseau (due perhaps to its having unfortunately been washed in the hard Oxford water instead of distilled water). One naturally puts réseau and photograph film to film, so that both may be in focus together; but when, as in this case, there is only a single exposure, spots in focus are easily mistaken for stars. For counting purposes, where one is not *measuring* from the réseau lines, this difficulty is got over by putting réseau and plate glass to glass, films outwards instead of together, and the spots are then so much out of focus as to be invisible, or readily distinguishable from stars, while the réseau is still in good enough focus for the purpose of defining a small area.

As a rapid way of estimating the number and distribution of star images, I counted only those in the diagonal line of réseau squares. I therefore mounted the plates diagonally in the counting microscope, so that the eye followed along a diagonal when the frame was moved horizontally.

I need not give the figures for each plate. The star-density curves, when reduced to percentages of the maximum density, are much the same for all of them. One plate, No. 50, was so much in the thick of the Milky Way (near the Southern Cross) that the number of stars—as many as 250 in one réseau square—quite swamped the other counts, and it would give a false impression to include it in ascertaining the *average* star-density, though it does not make much difference in the *form* of the density curve itself. The table below shows this. (The northern Milky Way plate happens to come on a poor region of the Milky Way, and the number of stars photographed, though well above the average, is not disturbingly abnormal.) The réseau lines are numbered from 0 at the centre to 16 at the corner, and the figures for equal areas on the sky are deduced from those for equal areas on the plate by multiplying them by the cube of the secant of the distance from the centre, given in the last column.

The diagram shows the result better. We see that up to about 9° from the centre the star density is fairly uniform, the undulations being doubtless due to incomplete compensation of the lenses, remarkably so at the centre of view. A sharp fall follows, and at 15° from the centre the density has fallen to about 50 per cent. of the maximum.

Here, then, we have a central area approaching to the uniformity of the doublet type of lens used for the C.P.D., apparently combined with the falling off beyond it of the single objective type; but with this difference, that whereas the curve required by Professor Turner's formula* is anticlinal, the curve of the Harvard plates is synclinal, and also the outer part is not a continuation by reversal of the inner part. (Compare Figs. 1 and 2.)

* P. 441 in *loc. cit.*

Réseau square.	Number of stars photographed at different distances from the plate centre on equal areas on the plate		on the sky		Star-density (Number per sq. deg.)	Distance from centre.
	Incl. pl. 50.	Excl. pl. 50.	Incl. pl. 50.	Excl. pl. 50.	Excl. pl. 50.	° ′
0 to 1	49·8	27·0	49·8	27·0	36·9	0 37
1 „ 2	56·0	30·9	56·0	30·9	42·2	1 50
2 „ 3	55·1	28·2	55·1	28·2	38·5	3 4
3 „ 4	53·5	30·0	54·0	30·3	41·4	4 17
4 „ 5	53·1	31·2	53·6	31·5	43·0	5 30
5 „ 6	52·3	30·9	53·3	31·5	43·0	6 43
6 „ 7	50·7	30·4	52·2	31·3	42·8	7 56
7 „ 8	52·4	29·8	54·5	31·0	42·4	9 8
8 „ 9	38·5	23·2	40·4	24·4	33·3	10 20
9 „ 10	34·2	20·6	36·3	21·8	29·8	11 30
10 „ 11	30·3	18·0	32·7	19·4	26·5	12 41
11 „ 12	27·5	18·0	30·0	19·6	26·8	13 50
12 „ 13	24·2	15·3	26·9	17·0	23·2	15 0
13 „ 14	21·1	13·7	23·8	15·5	21·2	16 8
14 „ 15	19·5	13·5	22·4	15·5	21·2	17 15
15 „ 16	15·7	11·0	18·4	12·9	17·6	18 23
Curve in diagram.	*a*	*b*	*c*	*d*	*e*	

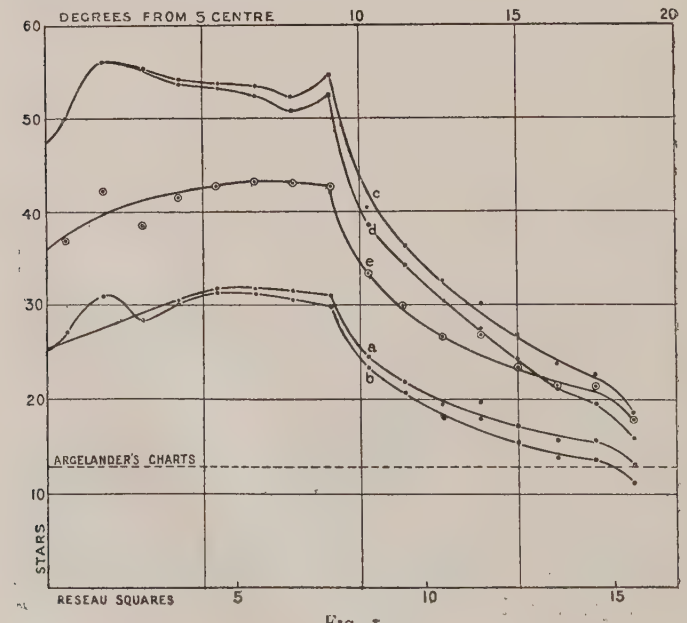


Fig 1

The change of star-density, once it begins, is more rapid with the doublet than with the single objective. Central uniformity is gained at the expense of the outlying parts, and it is possible that the Cape doublet would show a falling off of the same character were the field extended a few degrees further. The advantage of uniformity over the *whole* plate is obvious : it is the ideal of the lensmaker and the astronomer. The attractiveness of the C.P.D. plates in this respect is attained by confining the field of view within the limit indicated by the curves. The Harvard plates exceed this limit, but nevertheless it is satisfactory to find that the Harvard plates over their large area of $30° \times 30°$ have the same range of star-density (from 50 to 100 per cent.) as the astrographic plates over their area of $2° \times 2°$, with the additional merit of having the considerable area of 18° in diameter of practically uniform star-density.

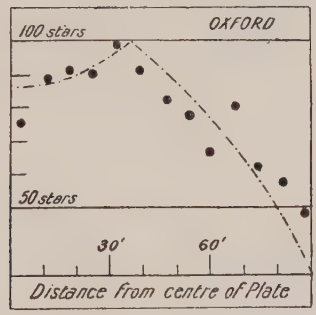


Fig. 2. (From Professor Turner's paper.)

It may be of interest to compare this star-density with Argelander's charts. To find this approximately, I drew circles on the charts a degree across at the plate centres and réseau corners, and laid down a pair of rulers touching them in pairs, centre to corner, and counted the stars between them. In this way I was counting approximately the same parts of the sky as I had counted on the photographs. The projection of Argelander's charts is not such as to make it exactly so, but it is near enough for the present purpose. I could, of course, only do this for the regions included in Argelander and Schonfield's charts, namely plates 1, 2, 6, 10, 16, 22, and 28. The count gave an average star-density of 12·9—less than the corner réseau square (which was not quite the corner of the plate). The mean Harvard star-density within the 18° circle is 41·3, so that the Harvard map, *over its area of reasonably uniform star-density*, shows nearly three times as many stars as Argelander, and is therefore comparable with the astrographic catalogue plates as well in respect to the magnitude of stars shown as to the relative star-density over the plate.

1903 *December.*

The Rotation Period of the Planet Saturn. By G. W. Hough.

The white spot first observed by Professor E. E. Barnard on June 15 was a conspicuous object during the month of July.

I secured an observation on June 27, but supposed for some time that it was another spot, since I imagined the rotation period would not differ very materially from that found by Professor Asaph Hall in 1877. Owing to the low altitude of the planet, combined with much cloudy weather and poor seeing, but few observations of spots were secured.

At my request Professor S. W. Burnham made micrometer measures of spots on *Saturn* with the 40-inch telescope of the Yerkes Observatory, which he has kindly communicated to me.

I found the Barnard spot had been observed on three nights by myself and on two nights by Burnham, covering a period of fifty-three days, or 119 rotations. I have also reduced a micrometer observation by Professor Barnard made on June 23 (*Astronomical Journal*, pp. 542, 543). The time given by the micrometer differs 6·8 minutes from that adopted by the observer as the most probable value.

It is to be regretted that astronomers generally have not used the micrometer to ascertain the time of passage of the spot over the central meridian of the disc in place of a single estimation. For about twenty minutes of time, when the spot is near the central meridian, the motion may be regarded as uniform, and the time of passage over the central meridian may be ascertained by micrometer measures without taking into account the size of the disc, the latitude of the object, or the rotation period.

At mean distance one minute of rotation time is equal to 0″·087 of arc. Hence one should expect a large accidental error from an eye estimate.

With the micrometer, on the contrary, one can make as many measures as is desirable, any one of which is of far greater value than an eye estimate.

Besides the principal spot there were others which were not so well defined.

The following notation is used :—

t = mean time of observation (six hours slow of Greenwich).

m = distance from the central meridian, reduced to mean distance.

Δt = reduction to the central meridian.

T = time of passage of the spot over the central meridian of the disc.

The constants for reduction were computed by using Barnard's value of the size of the disc, viz. $a = 8''{\cdot}90$, $b = 8''{\cdot}12$ (*A.N.* No. 3760).

In determining rotation period the observations have been corrected for motion in longitude, annual parallax, and aberration time.

Micrometer Measures, Barnard Spot.

Date 1903.	t h m	m "	Δt m	T h m	App. Disc. "	Obs.
June 23	15 03·0	+2·68	+32·2	15 35·2	+2·63	Bar.
27	14 45·3	+2·75	+33·0	15 18·3	...	...
	56·9	+1·86	+22·2	19·1	+2·74	...
Mean	...	...	...	15 18·7	...	Ho.
July 13	14 17·0	−0·13	− 1·6	14 15·4	...	...
	20·7	−0·47	− 5·4	15·3	+2·69	...
Mean	...	...	...	14 15·3	...	Ho.
18	12 11·0	−2·61	−31·3	[11 39·7]	+2·98	Ho.
29	13 27·0	−0·51	− 6·1	13 20·9	+2·90	β
Aug. 19	9 58·0	−2·24	−26·8	9 31·2	[+1·55]	β
	Minor Spots.					
July 6	13 38·9	−4·02	−49·5	12 49·4	+3·54	Ho.
22	12 42·2	−2·99	−36·2	12 06·0	+1·31	β
23	11 26·1	+0·13	+ 1·5	11 27·6	...	...
	34·2	−0·50	− 5·8	28·4	+0·68	β
Mean	...	...	...	11 28·0	...	...
23	foll. spot	+2·42	+29·0	11 57·0	...	β
30	11 20·0	+0·54	+ 6·5	11 26·5	...	β
Aug. 9	10 59·3	−0·11	− 8·3	10 51·0	...	Ho.
	foll. spot	+3·41	+41·6	11 32·6	...	Ho.
20	9 24·2	−2·44	−29·2	8 55·0	...	Ho.
20	9 31·0	+0·12	+ 1·3	9 32·3	...	Ho.

July 18.—Seeing very bad; approximate.
Aug. 19.—Spot faint and difficult.

By combining the observation of June 27 with the following ones successively we get for the rotation period—

d.	h m s
0 to 16	R = 10 38 19·0
0 „ 32	26·1
0 „ 53	29·0

These values indicate that the rotation period was not constant.

I have shown that the rotation periods for spots in the planet *Jupiter* are a function of the time, and are sensibly constant only for short intervals.

The following table shows the comparison of the observations with an ephemeris :—

First, for a uniform rotation period

$$R_1 = 10^h\ 38^m\ 27^s$$

and, secondly, for an increasing period

$$R_2 = 10^h\ 38^m\ 18^s + n \times 0^s{\cdot}1856$$

n = the number of rotations since June 27.

	h	m	s	s
$R_1 =$	10	38	27	...
$R_2 =$	10	38	18	$+n \times 0{\cdot}1856$

Date 1903.	Obs.	Days.	T.	$O-E_1$. m	$O-E_2$. m
June 23	Bar.	−4	15 35·2	+ 1·8	+0·4
27	Ho.	0	15 18·7	+ 0·0	−0·3
July 13	Ho.	16	14 15·3	− 4·8	−1·7
18	Ho.	21	[11 39·7]	[+15·5]	...
29	β	32	13 20·9	− 1·1	+1·4
Aug. 19	β	53	9 31·2	+ 4·1	−0·2

The constant period satisfies the observations with a mean residual $(O-E_1) \neq 2^m{\cdot}3$, or a probable error on rotation period $\neq 1^s{\cdot}1$.

For the variable period the mean residual $O-E_2$ is $\neq 0^m{\cdot}8$, which is better than one ought to expect from the difficulty of the observations.

The kronocentric latitude of the spot was deduced from the measures made on June 27 and July 13, viz. + 36° 33′ and +36° 22′ respectively.

The mean of the two values was adopted in the reduction of the observations.

The spots observed on July 6, 22, and 30 are identical. The rotation period for this spot is $10^h\ 38^m\ 30^s{\cdot}5 \neq 3^s{\cdot}2$.

Dearborn Observatory of North Western University,
Evanston, Illinois : 1903 *November* 11.

Observations of the Leonid Meteors of 1903 *made at the Royal Observatory, Greenwich.*

(*Communicated by the Astronomer Royal.*)

Until 4^h A.M. on November 16 meteors were appearing at a regular rate (about twenty per hour), and were easily kept under observation by Messrs. Showell and Parkinson, the two observers on duty. At 4^h, as there appeared to be no prospect of a shower, observations were suspended. Shortly afterwards an increase in the rate of appearance took place, and between 4^h 30^m and 5^h A.M. it was estimated by Mr. Furner that meteors were falling at the rate of about eighty per hour; a result closely confirmed by the regular observers on duty, who resumed observing from 5^h 54^m till 6^h 16^m A.M. During the night's watch (about five hours) the number of observations amounted to 107, nearly 90 per cent. of which were *Leonids*.

It should be added that Mr. Crommelin, who was on duty with the altazimuth on November 15 from 4^h 30^m to 6^h 0^m (civil time), and was looking at the sky fairly continuously during the time, saw no meteors.

The morning of November 17 was cloudy.

Royal Observatory, Greenwich:
1903 *December* 8.

The Shower of Leonids in 1903. By W. F. Denning.

A strikingly abundant display of these meteors occurred on the morning of Monday, November 16. Watching the clear north-eastern sky between 12^h and 14^h (November 15) I found the horary number of *Leonids* about eighteen for one observer, while from 14^h to 16^h the rate increased to fifty. Between 16^h and 18^h there was a further rise to about 130 per hour. The maximum was from 17^h 30^m to 17^h 45^m, when forty-two were counted. Between 17^h 30^m and 18^h 5^m the rate of apparition must have exceeded 200 per hour for one observer watching the sky from a good open position. Here the view is somewhat restricted by trees and buildings, so that the meteors enumerated were certainly less than the numbers actually visible under the best circumstances. The shower was about five times as rich as a normal display of *Perseids*.

I counted the meteors at alternate intervals of 15^m, so that I might not only be able to trace the strength of the shower, but also to record some of the brighter objects and note special features. The following is a summary of the chief results.

The meteors were bright generally, and nearly all 1st and 2nd mags. Few, however, were of sufficient brilliancy to outshine *Venus*. Perhaps about one in twenty-five appeared as bright as *Jupiter*, while one in 100 was equal to *Venus*. In the showers of August *Perseids* and November *Andromedids* a considerable number of very small meteors are intermingled with the more brilliant members, but in this respect they offer a marked distinction to the *Leonids*.

The radiant was diffused over an area of about 6°, though a large proportion of the flights were accurately directed from a central point at 151°+22°. Many short and well-observed paths in and near the sickle showed the dispersion very distinctly from R.A. 147° to 155° and Dec. +20° to 26°.

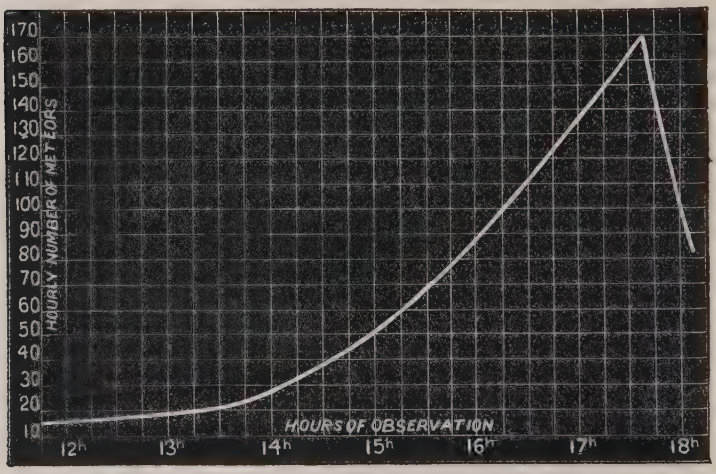


FIG. 1.—Hourly number of meteors seen by one observer. 1903 November 15, 12^h to 18^h.

In the majority of instances the apparent motions were "very swift" or "swift," but the velocities varied in a manner which could hardly be explained by differences in the individual distances or in the angles under which the lines of flight were presented. Several *Leonids* seen here when the radiant was low were described as "slow" or "rather slow," and if at ordinary heights, they could scarcely have travelled with velocities exceeding 30 or 35 miles per second, owing probably to the effects of atmospheric resistance.

The really active portion of the shower was confined to the morning of November 16. On the previous morning I watched a beautifully clear sky for three quarters of an hour (November 14 17^h to 17^h 45^m); but though "the Sickle" was high in the south not a single meteor shot from it, though five were observed from

other directions. On the morning of November 17 the firmament was not very favourable, but during occasional observation

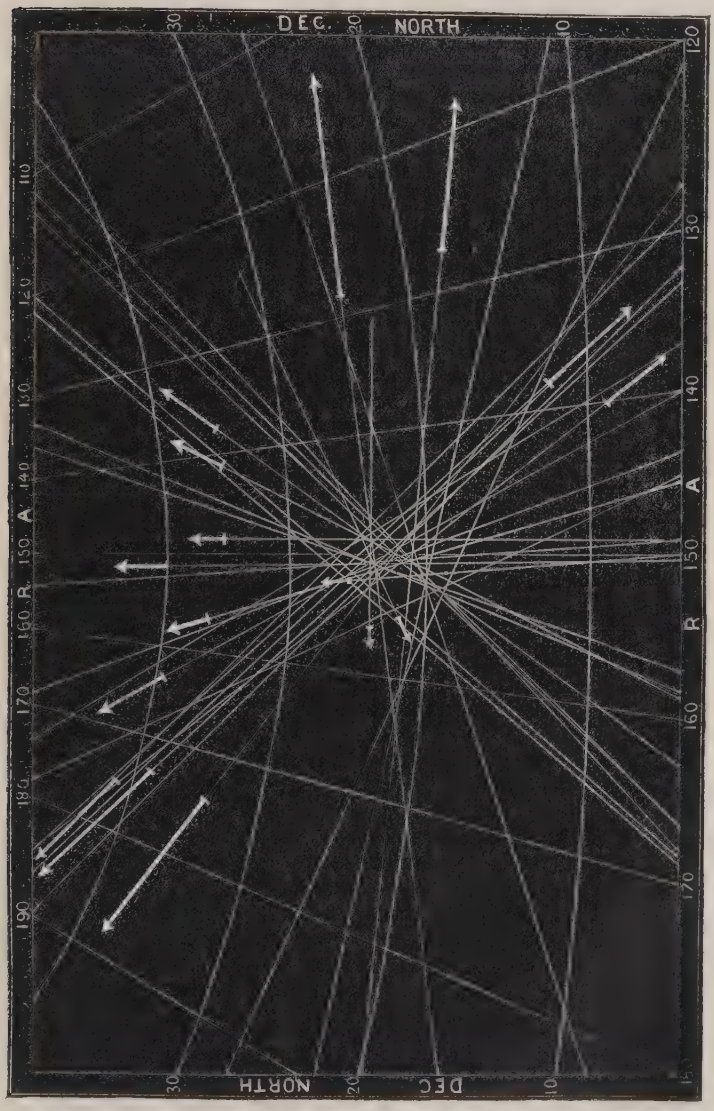


FIG. 2.—The Radiant of the *Leonid* shower, 1903 November 15, 12h to 18h

meteors were found to be rare, while the *Leonid* shower afforded only scanty evidence of its presence.

Taking the maximum as having occurred on November 15, $17^h\ 40^m$, the Earth's longitude was 52° 41′, while the bright shower observed in America in 1901 reached a maximum on November 14 at about $22\frac{1}{2}^h$ G.M.T., when the Earth's longitude was 52° 23′. The latter shower appears to have been decidedly stronger than the former, for at the richest time in 1901 various observers found the rate of appearance six or seven per minute. In 1903, during the half-hour when meteors were falling in most plentiful numbers, the rate, as determined under the best surrounding conditions, was about four per minute.

As usual in such displays the meteors came in bursts : two or three were frequently noticed at the same moment pursuing parallel flights in nearly the same region. During the most active period there were successive volleys of four, five, or six. At $16^h\ 45^m$ two bright flashing *Leonids* (one = *Jupiter*, the other > *Venus*) were seen almost at the same instant : one crossed the head of *Auriga* at δ, the other left a streak for more than a minute on the fore paws of *Leo* between *o Leonis* and ζ *Hydræ*.

I observed twenty-seven meteors during the night belonging to the numerous minor showers of the epoch. An exceedingly slow red meteor, of mag. 4, was recorded, at $15^h\ 41^m$, struggling along a short arc from 219°+63° to 209°+61¼° in about three seconds. It very probably belonged to a long-enduring radiant at 262°+62° near ζ *Draconis*. At $15^h\ 59^m$ an unusually slow streak-leaving meteor of mag. 2 traversed a long flight of 45° from 178°+19° to 223°+41° (approximately from β *Leonis* to β *Boötis*). Its visible course may have been even longer than this, for it sailed into my view from behind the corner of a house. The meteor was also seen by Mr. W. Parkinson at the Royal Observatory, Greenwich ; by the Rev. S. J. Johnson at Bridport, Dorset; and by Mr. F. H. Wright, Northampton; and its radiant appears to have been low in the south-eastern sky at 147°−11°. A preliminary determination of the real path indicates the flight, as an extended one of 128 miles, from over Kent to Lincoln, at a height gradually descending from 91 to 45 miles. Velocity about 29 miles per second. At the end of its flight the meteor slowed up considerably, and became almost stationary, as though, its material being well nigh spent, the increasing pressure of the atmosphere had effectually retarded its further progress. Additional observations of this interesting object would be valuable. Its individuality of direction and slow, sailing flight over an extensive arc of the heavens must have caused it to arrest the special notice of many persons who were watching the *Leonid* shower.

Duple or multiple observations of several bright *Leonids* were obtained during the recent display, and the real paths of these will be computed. A 2nd mag. *Leonid* was seen on November 15 $13^h\ 18^m$ by Mr. C. L. Brook at Meltham, near Huddersfield, and also by the writer at Bristol. It fell from a height of 73 to 58 miles, along a path of 36 miles, at a velocity

of about 48 miles per second. The radiant was at 151°+24°. Mr. Brook has also calculated the real path, and makes the height 77 to 61 miles, length of course 34 miles, and velocity 45 miles per second.

For comparison with this and other similar results it may be useful to give the average heights, &c., of seventeen *Leonids* doubly observed in England during the last seven years.

Height at first appearance	84·1 miles
„ disappearance	55·9 „
Length of visible course	45·1 „
Velocity per second	49·8 „
Radiant point	150·65+23·1

Subjoined are the apparent paths of a few bright *Leonids* and of several other shooting stars recorded here during the recent display, and it would be interesting to have duplicate observations of any of these objects.

Date. 1903	G.M.T.	Mag.	Path From α	Path From δ	Path To α	Path To δ	Length.	Duration.	Remarks.	Probable Radiant.
	h m		°	°	°	°	°	sec.		
Nov. 15	12 36	1	122	+24½	103	+23½	17½	1·6	slow, stk.	L.
	12 53	1½	187½	+44	211	+48	17	1·4	slow, stk.	L.
	13 9	3	265	+70	245	+63	10½	1·0	slow	332+71 / 13+39
	13 13	1	119½	+48	97	+53½	15	0·8	v. swift, stk.	L.
	13 35	5	129	+26½	138	+28	8	0·9	slow	Orion
	13 40	3	130	+60	147	+59	8½	1·0	slow	α Aurigæ
	14 32	1½	141½	+35	138½	+39	5	0·5	swift, stk.	L.
	14 36	>1	117½	+26½	98	+26¼	18	0·9	swift, stk.	L.
	14 41	2	32	+71	2	+81	12	0·7	swift	48+43
	14 43	1	162	+38½	168	+44	7	0·6	swift	L.
	14 43	4	73	+72½	17	+63	22	1·2	swift, stk.	L.
	15 9	5	227	+60½	234	+57½	5	0·9	slow	194+67
	15 13	1½	173½	+34½	187	+38½	11½	0·7	slow, stk.	L.
	15 38	4	134	+58½	147	+49	12	1·2	slow	13+39
	15 41	4	219	+63	209	+61¼	5	3·0	v. v. slow	262+62
	15 59	2	178	+19	223	+41	45	5·0	v. v. slow	147−11
	16 4	1	168½	+37	186	+45	15	1·0	swift, stk.	L.
	16 12	4	70	+70	19	+59	23	1·3	swift, stk.	L.
	16 35	2	202	+62	217	+46½	17½	1·3	swift	α Aurigæ
	16 37	4	138	+35	133	+39	6	0·5	swift, stk.	L.
	16 45	4	92	+53	68	+52½	15	0·9	swift, stk.	L.

Date. 1903.	G.M.T.	Mag.	Path From α	Path From δ	Path To α	Path To δ	Length.	Duration.	Remarks.	Probable Radiant.
	h m		°	°	°	°	°	sec.		
Nov. 15	16 45	♀	138	+12	134	+7½	6	0·5	swift, stk.	L.
	17 7	4	165½	+41	161½	+36	6	0·5	swift, stk.	177+49
	17 11	1	213	+29	223	+27	9	0·7	swift, stk.	L.
	17 14	4	173	+40½	190	+47	13½	0·9	swift, stk.	L.
	18 5	>4	182	+51	195	+55½	9	0·7	swift, stk.	L.
16	5 59	>1	343	+38½	308	+34	29	3·5	v. slow, yellow train	42+20 58+9

Fig. 1 exhibits the increase in the hourly number observed. This was due to two causes—viz. the much greater density of that part of the stream through which the Earth passed towards 18^h and the improving position (higher altitude) of the radiant as the night progressed.

In fig. 2 the diffuse character of the radiant is shown by the intersecting lines of flight of a number of well-observed *Leonids*. In some cases the meteors appeared outside the limits of the diagram, but their paths have been carried backwards through the radiant.

Bishopston, Bristol: 1903 *November* 25.

Cape Double Star Results, 1903. By R. T. A. Innes.

(*Communicated by Sir David Gill, K.C.B., F.R.S., H.M. Astronomer.*)

The observations in 1903 were confined to a few nights in February and March.

All the measures have been made with the Repsold micrometer on the 18-inch refractor. The assumed value of one revolution of the micrometer screw has been taken as $17''{\cdot}600$.

An explanation of the columns headed "Diff. of Mags." and "Colours" is given on p. 471 of the *Monthly Notices* for 1902 April, vol. lxii.

My measures of double stars with the above instrument extend altogether from 1899 November to 1903 March, and are now brought to a close by my transfer to the Transvaal. My thanks are due to Sir David Gill for his constant encouragement and help, and for the clerical assistance placed at my disposal. The work was greatly facilitated by the mechanical conveniences of the McClean Observatory, the rising floor in particular being of the utmost value for double-star work.

Star.	Mag.	1900. R.A.	1900. S. Dec.	1902+	Pos. Angle.	Dist.	Diff. of Mags.	Colours A	Colours B	No. of Nights.	Remarks.
		h m	° ′		°	″					
Innes 387 ...	8·0	3 1	60 23	16	142·0	0·51	0·0	2, 2		1	
„ 388 ...	8·3	3 14	63 39	16	314·5	0·62	0·2	...		1	
Cape 38 ...	9·0	3 42	40 29	16	211·7	3·96	0·5	...		1	
h 3642 ...	6·7	4 15	34 9	16	159·8	6·05	2·0	2, *b*		1	
Innes 413 ...	7·8	4 23	24 41	16	349·8	0·80	0·8	...		1	
h 3683 ...	6·6	4 39	59 8	16	80·6	1·54	0·1	3, 3		1	1
Cordoba [9]...	7·4	4 39	48 1	16	234·8	3·73	2·0	2, *b*		1	2
„ [10]...	7·5	4 46	50 3	16	258·4	4·82	2·5	2, *b*		1	
Innes 342 ...	7·5	4 47	54 4	16	175·5	0·91	0·5	3, 3		1	
„ 343 ...	7·8	4 48	54 37	16	45·6	2·56	3·5	...		1	
λ 42	6·9	4 49	51 54	16	Single	...	...	...		1	
h 3707 ...	9·2	4 51	59 55	16	270·6	8·50	3·0	...		1	
Innes 274 ...	8·0	5 0	50 56	16	259·4	3·34	1·7	2, *b*		1	
Cordoba [12]	8·4	5 2	40 42	17	241·8	2·14	0·4	1, 1		2	
h 3743 ...	8·5	5 12	60 6	17	131·4	5·11	1·7	*b*, 2		2	
Cordoba [13]	8·8	5 14	27 36	16	274·0	3·21	0·4	...		1	
Innes 61 ...	7·8	5 15	41 9	22	100·5	0·91	0·5	1, 1		1	
„ 414 ...	6·4	5 21	56 15	19	Single	...	...	...		2	3
θ Pictoris ...	6·3	5 23	52 24	19	194·3	0·48	0·3	3, 3		2	
Innes 346 ...	5·9	5 24	41 2	19	180 ±	15 ±	8·1	...		1	4
h 3770 ...	7·6	5 29	24 24	22	16·4	4·17	3·2	3, *b*		2	
Innes 278 ...	8·5	5 43	68 45	19	174·2	1·20	2·5	...		2	
„ 279 ...	8·5	5 43	43 18	20	2·0	0·75	1·8	3, 3		3	
„ 63 AB	7·1	5 46	48 57	22	12·5	1·19	1·8	...		1	
„ „ AC				22	209·6	32·3	6·5	...		1	
h 3807 ...	8·3	5 49	41 42	16	270·8	5·15	2·7	2, *b*		1	
Cape 39 ...	9·4	6 19	73 26	17	259·9	5·49	1·1	...		2	
G Puppis ...	6·0	6 23	48 7	16	132·5	1·30	3·0	2, *b*		1	
Innes 350 ...	7·5	6 26	45 43	22	10·6	15·9	5·0	...		1	
Lac. 2333 AB	5·7	6 27	50 10	19	261·1	0·64	0·1	$3\frac{1}{2}$, $3\frac{1}{2}$		2	
„ CD	6·7			19	117·8	0·40	0·0	$6\frac{1}{2}$, $6\frac{1}{2}$		2	
β 754 ...	7·0	6 31	33 56	19	36·3	0·87	0·4	3, 3		2	
Innes 178 ...	8·3	6 31	32 14	19	203·1	0·80	2·0	...		2	
C.Z. 1518 ...	7·6	6 33	48 12	19	201·2	10·7	1·8	$2\frac{1}{2}$, *b*		2	
Dunlop 31 ...	5·0	6 36	48 8	16	319·2	13·5	2·0	4, *b*		1	
Innes 180 AB	8·5	6 42	52 19	16	336·5	0·81	1·0	...		1	
„ 157 ...	7·0	6 45	54 35	16	343·6	1·92	3·0	2, *b*		1	
„ 158 ...	7·2	6 45	48 27	22	184·0	1·69	3·5	3, *b*		1	

Star.	Mag.	1900. R.A.	1900. S. Dec.	1902+	Pos. Angle.	Dist.	Diff. of Mags.	Colours A	Colours B	No. of Nights.	Remarks.
		h m	° ′		°	″					
Innes 181	8·0	6 47	44 56	22	255·0	0·68	1·0	0,	0	1	
„ 182	8·0	6 47	28 36	22	136·3	0·79	2·0	0,	0	1	
„ 431	8·5	6 47	28 37	22	321·5	0·37	0·0	...		1	5
„ 432	7·8	6 53	28 36	22	213·2	1·65	0·0	...		1	
„ 65	6·3	6 54	35 23	18	216·3	0·30	0·0	$3\frac{1}{2}$,	$3\frac{1}{2}$	3	6
Innes 184	8·0	7 8	60 25	18	179·3	0·71	0·4	...		3	
Harvard	6·0	7 11	63 1	19	153·2	0·53	0·4	2,	2	2	
Innes 7	7·3	7 15	46 49	22	212·0	1·10	...	4,	*b*	1	
„ 391 AB	8·7	7 16	21 42	16	275·2	4·67	0 2	...		1	
„ AC				16	30 ±	5 ±	4·3	...		1	
Innes 312	7·7	7 19	75 41	22	166·9	0·88	0·0	3,	3	1	
„ 352	8·5	7 22	37 57	19	356·6	0·67	0·6	0,	0	2	
h 3997	6·1	7 37	74 3	16	293·2	1·65	0·1	2,	2	1	
h 4003	9·2	7 44	23 56	18	117·0	31·2	1·7	$\frac{1}{2}$,	6	2	7
Innes 161	5·3	7 44	38 16	18	84·5	10·6	4·0	1,	*b*	1	
Cape 20	8·5	7 45	44 18	18	88·1	3·49	1·0	...		1	
ξ Argûs	3·4	7 45	24 37	18	241 ±	...	...	...		1	8
Howe 8	5·0	7 49	34 27	18	289·2	2·94	3·0	$2\frac{1}{2}$,	*b*	2	
β 333	7·7	7 57	22 4	18	45·1	1·83	2·5	...		2	
Howe	8·5	7 58	26 59	18	331·1	2·49	2·5	...		1	
h 4041	7·8	8 1	22 9	17	Single	...	...	...		1	
Lac. 3146 AB	6·5	8 2	33 17	20	87·4	22·3	2·0	4,	*b*	2	
„ AC				20	58·3	27·9	4·0	...		2	
Innes 164	7·5	8 3	39 49	22	83·9	0·62	1·0	...		1	
„ 193	8·1	8 10	34 26	17	103·8	2·48	1·5	...		2	
Howe 9	8·7	8 13	26 58	17	118·1	3·26	0·1	0,	0	2	
Cordoba [77]	8·2	8 29	50 38	18	99·3	3·53	1·3	3,	*b*	2	
Innes 68	8·6	8 33	30 29	17	67·2	7·49	2·0	...		2	
Cordoba [19]	8·0	8 37	31 27	18	328·8	5·49	2·7	3,	*b*	2	
Innes 315	8·5	8 38	42 14	18	166·8	0·99	0·2	...		1	
„ 70	7·0	8 44	38 34	18	111·9	1·58	2·0	...		1	
„ 317	8·5	8 52	43 5	18	304·3	2·19	1·5	1,	*b*	1	
h 4164	7·5	8 56	65 49	18	145·9	10·7	2·5	4,	B	1	
b_2 Carinæ	5·2	8 57	58 42	18	20·8	28·2	...	...		2	
Innes 357	8·4	8 58	23 21	24	177·1	0·67	0·1	...		1	
h 4165	5·4	8 59	51 48	24	103·6	1·46	2·0	2,	—	1	
Innes 319	9·1	9 10	82 37	18	86·2	4·47	1·0	...		1	
„ 11	6·5	9 12	45 8	22	271·7	0·91	1·5	...		1	

Star.	Mag.	1900. R.A.	1900. S. Dec.	1902+	Pos. Angle.	D'st.	Diff. of Mags.	Colours A	Colours B	No. of Nights.	Remarks.
		h m	° ′		°	″					
Innes 358	... 5·4	9 16	68 16	18	133·0	18·1	7·1	3, —		1	
Cape 33	... 8·4	9 16	69 58	20	339·0	4·76	0·6	2½, 0		2	
Innes 198	... 8·0	9 16	28 48	22	178·9	0·33	...	...		1	
„ 170	... 7·5	9 17	34 56	22	32·4	1·31	3·0	...		3	
„ 12	... 5·4	9 18	74 28	22	263·3	0·31	0·3	2½, 2½		2	
„ 72	BC 9·6	9 23	52 57	21	32·9	3·14	1·0	...		1	
„ 199	... 7·8	9 23	69 59	18	136·1	2·69	2·5	0, *b*		1	
ψ Argus	... 3·5	9 27	40 2	20	24·1	0·29	...	3, 3		6–5	
Innes 203	... 8·2	9 36	62 6	22	313·8	0·45	0·3	...		1	
Hargrave 28	... 7·8	9 42	59 1	22	284·6	2·02	2·5	3, B		1	
Innes 204	... 7·2	9 44	69 39	16	127·7	1·87	2·5	...		1	
„ 174	... 8·4	10 28	61 4	16	46·0	1·06	0·5	...		1	
Jacob [8]	... 6·4	12 5	34 9	23	21·2	3·54	...	2, *b*		1	
Harvard AB	... 6·2	12 23	55 51	23	275·4	27·2	6·8	...		1	
AC	...			23	290·2	48·9	5·8	...		1	
γ Centauri	... 2·4	12 36	48 25	22	353·6	1·42	0·0	3, 3		3	
Cordoba [27]	8·3	12 39	61 40	23	98·1	4·80	...	6, *b*		1	
Harvard	... 6·7	12 47	53 17	23	210·6	6·05	5·0	4½, —		2	
Innes 399	... 7·5	13 8	62 7	23	137·6	8·32	4·0	3, —		1	
„ 400	... 8·8	13 22	42 41	23	151·5	4·75	3·0	3, *b*		1	
h 4598	... 7·0	13 33	74 37	23	46·5	13·1	4·0	3, —		1	
h 4616	... 9·5	13 45	70 41	23	349·5	4·55	1·0	...		1	
Harvard	... 5·2	14 8	56 37	23	169·3	33·6	...	...		1	
„	... 5·9	14 9	66 7	23	307·0	24·1	7·1	...		1	
Dunlop 159	AB 5·0	14 15	58 0	18	160·2	9·51	2·0	4, B		1	
	AC			18	255·0	20·7	8·5	...		1	
	AD			18	4·0	45·6	5·5	...		1	
Innes 426	... 5·6	14 24	44 52	18	313·6	10·5	5·0	2, —		1	
Innes 367	... 8·0	14 32	64 17	18	357·6	4·15	4·0	3, —		1	
α Centauri	... 0 2	14 33	60 25	21	211·2	21·66	1·2	2·3, 4		10	
α Circini	... 3·4	14 34	64 32	20	236·0	15·8	4·0	2, 7½		2	
Innes 334	... 9·0	15 54	71 37	23	141·0	2·53	1·5	2, *b*		1	

Remarks.

1. Fixed in angle; some decrease in distance.
2. P.M. of 0″·094 towards 238°·1.
3. To be rejected as single.
4. Too faint to measure.
5. New. The chief star of the old pair *h* 3896.

6. The angle increases 10° a year.
7. Mag. from Cor. D.M. Estimated 8·0. Is 8·7 in C.P.D. and marked: Angle decreasing, distance decreasing.
   There is a 13th mag. star 200° ±, 35″ ±.
8. Change. Two careful estimations with the chief star out of the field.

Short Method for the Calculation of the Orbits of Celestial Bodies.

By D. A. Pio.

(*Communicated by the Secretaries.*)

1. *Gain of Time is the Object.*—The determination of the elements of the orbits described by a celestial body has been carried by Gauss to the highest pitch of perfection the present state of mathematical science allows us to reach. The author has not the pretension to propose a new method for solving the same problem; he only intends to substitute short, simple, and easy calculations for those which the method of Gauss requires, and limits himself to the most simple case: how to determine the first approximate values of the elements of a newly discovered celestial body with the least loss of time, so as to render the calculation of a new orbit an easy performance.

On the authority of the illustrious French mathematician M. H. Poincaré, the writer begs to state that the calculation of the first approximate orbit, by Gauss's method, requires from fifteen to seventeen hours. A reduction to about six to seven hours is what the author is working for. A gain of about eleven hours with a sufficiently approximate determination of the elements would be a service to practical astronomers.

The author proposes, for attaining this short calculation of orbits, a modification of Gauss's method containing innovations of great practical interest for the efficient simplification of the mathematical work.

The writer begs to remark that the problem of orbits has been really solved long ago; that the question is only how to render tedious calculations more easy, but not to present new theories by which the work of La Place, Olbers, and Gauss may be swept away. This is the point of view from which the author begs his work to be considered. Practical improvement, not radical subvertment, is the only amelioration Gauss has left posterity to accomplish.

2. *Preliminary Remarks.*—It is supposed that the data furnished by the observations and those taken from the solar tables are corrected for precession, nutation, and aberration of the fixed stars. Parallax and aberration in time are not taken into consideration. The calculations are made with logarithms to not more than five decimal places.

3. *Simplifications of Gauss's Method.* – (1) Gauss established the equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

which leads to the value of r_2. This part of the solution cannot be improved. However, modern authors have brought confusion into the signs (+ or −) of the many auxiliary quantities occurring in the calculation: especially the case where r_2 is *less* than unity offers difficulties. The writer has reverted to the *Theoria Motus*, and has removed all ambiguities (see articles 4 and 5).

(2) The solution of the equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

has been rendered shorter by using logarithms with three decimals for the first approximations. The method of logarithmic differences has been reduced to its simplest form (see articles 7 and 8)

(3) Gauss deduces the elements of the orbit from the known values of r_1 and r_3, and from the angle $2f' = v_3 \sim v_1$ contained between them. The calculation of these quantities requires the previous calculation of the distances from the Earth, ρ_1 and ρ_3, which as done in the usual manner is the most lengthy and the most tedious part of the whole work. The usual formulæ, as given in Tisserand, *Leçons sur la Détermination des Orbites*, pp. 76 and 77, are twenty-four in number, and some are very troublesome. The writer has reduced them to six by using the "fundamental equations" for the calculation of $r_1 = \rho_1 \cos \beta_1$ and $r_3 = \rho_3 \cos \beta_3$. The time thus gained amounts to some hours (see article 9).

(4) The parameter, the eccentricity, and the longitude of the perihelium of an orbit are the most difficult points in the determination of the elements. Gauss invented a most ingenious method for the calculation of the parameter by successive approximations through auxiliary tables. The author has much shortened the calculation of the parameter by reverting to the formula of Euler given in *Theoria Motus*, § 86 (VII.). It is the shortest existing.

(5) The formulæ usually employed for the calculation of ϕ, the angle of eccentricity, and the true anomalies v_1 and v_3 are very tedious. They occasion a considerable loss of time (see Tisserand, p. 80). The arc v_1 has been determined by a simple formula taken from *Theoria Motus*. The value of the eccentricity may be calculated very easily when the parameter, one of the radii vectores, and the corresponding true anomaly are known. The author, profiting by this circumstance, calculates *two* separate values for e and takes their *geometrical mean* as the most probable value.

Each of the above modifications is certainly a detail of no theoretical importance by itself, and has no other merit than

either to shorten the calculation or to increase the accuracy. The author begs to submit that all these modifications, taken as a whole, give quite a new appearance to the method of Gauss.

The question is, then, whether the above modifications satisfy or not the requirements of practical astronomers.

4. *Rules of Signs.*—The calculations preliminary to the solution of

$$\mathfrak{M} \,.\, \sin^4 z = \sin (z + \omega)$$

are relative to the determination of the quantities ψ, δ, τ, T, Σ, σ, S, d, u, ω, Ω, and h. The angles ψ, δ, τ, and u are to be taken always as positive arcs *less* than 180°. The arc σ is always less than 90° : it may be positive or negative, but it *must* be the *least* of all values that result from the absolute number found for $tg\ (\sigma + \delta)$; and the sign of $tg\ (\Sigma = \delta + \sigma)$ must be determined in conformity. The arc ω must always be less than 90°, but it may be negative or positive. The signs of the linear quantities T, S, d, Ω, and h are + or − according as the calculation determines them. Every other rule, even given by great authorities, leads to impossible results.

Note.—Oppolzer's rule to make T always positive is false when r_2 is less than 1. In this last case ω also is positive and not negative.

5. *A Difficulty in the Solution of* $\mathfrak{M} \,.\, \sin^4 z = \sin (z + \omega)$.—The equation

$$\mathfrak{M} \,.\, \sin^4 z = \sin (z + \omega)$$

is usually applied only to the planetoids where r_2 is comprised between 1·5 and 4, and then ω is usually negative and nearly equal to z. In such cases the unknown value of z is found by successive approximations, the value of $-\omega$ being the first, and the method of logarithmic differences leads very rapidly to a value sufficiently approximate for the first determination of the orbit. However, when r_2 is less than R_2, then δ is greater than 90°, and the angle ω is not only positive but very different from z, so that it becomes difficult to find a first approximation for z. As usually ω is small, by assuming $\sin \omega = 0$ the equation

$$\mathfrak{M} \ \sin^4 \omega = \sin (z + \omega)$$

becomes

$$\mathfrak{M} \,.\, \sin^3 z = \cos \omega$$

and then

$$\sin z = \sqrt[3]{\frac{\cos \omega}{\mathfrak{M}}}$$

which may serve as first approximation.

6. *Logarithms to three Decimal Places.*—In the usual manner of calculating the orbits much time is wasted by using logarithms to many decimal places, while the results found out with so

much labour are scarcely trustworthy to the third figure. This happens with the first attempts to solve the equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

Also the correction of the approximate value of the parameter (Euler's formula) needs only logarithms to three decimal places.

7. *Logarithmic Differences.*—The method of logarithmic differences given by Gauss in *Theoria Motus*, § 11, for the solution of Kepler's Problem is excellent for solving the equation

$$\mathfrak{M} \cdot \sin^4 z = \sin(z+\omega)$$

Its shortest form is that the author proposes.

EXAMPLE.—$N[1{\cdot}0822321] \cdot \sin^4 z = \sin(z-12°)$, where $\mathfrak{M} = N[1{\cdot}0822321]$, $\omega = -12°$, λ = logarithmic difference of $\sin z_2$ for one minute (respectively one second), μ = log dif. for 1′ (or 1″) of $\sin(z_2+\omega)$, $\Delta = \log \sin^4 z_2 - \log \sin^4 z_1$, $z_1 = -\omega$.

$\log \sin z_1$	9·318	$\lambda = 0{\cdot}5$	$\log \sin^4 z_2$	9·362
	4			4
$\log \sin^4 z_1$	7·272	$\mu = 5{\cdot}6$	$\log \sin^4 z_2$	7·448
$\log \mathfrak{M}$	1·082	$\mu - 4\lambda = 3{\cdot}6$	Δ	
			$\overline{\mu - 4\lambda}$ =	49′
$\log \sin(z_2+\omega)$	8·354	$\Delta = 176$		
$z_2+\omega = 1° 17'{\cdot}5$		$\omega = -12°$	$z_2 =$	13° 17′·5
			$z_3 =$	14° 6′·5

Six lines with logarithms are sufficient.

8. *Calculation of* $\rho_1 \cdot \cos\beta_1$ *and* $\rho_3 \cdot \cos\beta_3$.—The condition that the centre of the Sun must lie in the plane determined by the three observed positions of the celestial body furnishes three "fundamental equations," (5), (6), (7), in Tisserand, p. 47, containing the unknown quantities $r_1 = \rho_1 \cdot \cos\beta_1$, $r_2 = \rho_2 \cdot \cos\beta_2$, $r_3 = \rho_3 \cdot \cos\beta_3$. One of them, r_2, being known, the determination of r_1 and r_3 becomes very easy. Equation (7) furnishes directly r_1, and from (6), where r_1 and r_2 are known, r_3 is easily found.

9. *Concluding Remarks.*—(1) The abridgments introduced by the author will furnish the values of the elements with sufficient accuracy, when the intervals of time between two successive observations are not more than ten days for planetoids.

(2) The purpose of a first determination of an orbit being first the calculation of a provisory ephemeris, and next the setting-up of numerical values for the elements approximate enough to allow the easy correction of these numerical values by comparison with a series of observations, any luxury of calculations, as those indicated in the text-books, is a mere waste of time. Whatever be the care bestowed upon the first determination of an orbit, the result arrived at by so much work will be *discarded*. Therefore brevity ensuring sufficient accuracy is the ideal of the first determination of an orbit.

(3) The calculation of an orbit is considered as the trial-piece of an incipient astronomer. It is, indeed, a proof of proficiency, especially if carried out by the method of Gauss, as explained in Oppolzer's text-book. But even the dreaded method of Gauss may be popularised, and that is what the author has attempted. In about six to seven hours, when the observations have been already reduced and corrected, to determine with sufficient accuracy the numerical values of the elements of an orbit without employing special auxiliary tables, only using the common logarithmic tables of numbers and trigonometrical functions to no more than five places of decimals, this is certainly no achievement in theory, but only a sort of mechanical craft. However, is it useful or not? Has the writer succeeded in building up an engine of calculation proper for the use of practical astronomers?

10. *A Test for Asteroids.*—The writer has chosen as an example for the application of the simplified method of Gauss the same observations from which Gauss himself deduced the orbit of *Juno* in *Theoria Motus*, § 150 to § 155. His calculations are a model of accuracy, and a comparison with those made by the writer shows the degree of precision the first values for the elements may reach when the modifications proposed by the author are used. The following table contains in its first column the results of the "last hypothesis" of Gauss :

	Gauss.	Pio.	Differences.
r_2	2·1183	2·1187	+0·0004
$2f'$	7° 34′ 54″	7° 35′ 7″	+13″
a	2·6450	2·6499	+0·0049
e	0·24532	0·24744	+0·00212
i	13° 6′ 44″	13° 7′ 30″	+46″
Ω	171° 7′ 49″	171° 8′ 29″	+40″
Π	52° 18′ 9″	52° 30′ 23″	+12′ 14″
m''	824″·80	822″·50	−2″·3
L_0	41° 52′ 22″	41° 58′ 22″	+6′ 0″

Formulæ.

1. *Symbols.*—For the three observations of the planet at the epochs E_1, E_2, and E_3, we have :

$E_2-E_1=t_1$.
$E_3-E_2=t_2$.
R_1, R_2, R_3, radii vectores of Earth.
L_1, L_2, L_3, geocentric longitudes of Sun.
λ_1, λ_2, λ_3, geocentric longitudes of planet.
β_1, β_2, β_3, geocentric latitudes of planet.
l_1, l_2, l_3, heliocentric longitudes of planet.
b_1, b_2, b_3, heliocentric latitudes of planet.

r_1, r_2, r_3, radii vectores of planet.
v_1, v_2, v_3, true anomalies of planet.
ρ_1, ρ_2, ρ_3, distances of planet from Earth.
e, eccentricity of planet.
p, half parameter of planet.
a, half major axis of planet.
T^d, planet's revolution in mean solar days.
m'', its mean diurnal motion in seconds of arc.
i, inclination of planet's orbit.
Ω, longitude of planet's ascending node.
Π, longitude of planet's perihelium.
L_o, mean longitude of planet at epoch E_o.
E_o, epoch of departure.

2. *Preliminary Calculations.*—

$$A = R_1[tg\beta_3 . \sin(\lambda_1 - L_1) - tg\beta_1 . \sin(\lambda_3 - L_1)] \quad \dots \quad (1)$$

$$B = R_3[tg\beta_3 . \sin(\lambda_1 - L_3) - tg\beta_1 . \sin(\lambda_3 - L_3)] \quad \dots \quad (2)$$

$$C = R_2[tg\beta_3 . \sin(\lambda_1 - L_2) - tg\beta_1 . \sin(\lambda_3 - L_2)] \quad \dots \quad (3)$$

$$D = -R_2[tg\beta_3 . \cos(\lambda_1 - L_2) - tg\beta_1 . \cos(\lambda_3 - L_2)] \quad \dots \quad (4)$$

$$tg . \psi = \frac{tg\beta_2}{\sin(\lambda_2 - L_2)} \quad \psi \text{ positive and } < 180^\circ \quad \dots \quad \dots \quad (5)$$

$$tg\delta = \pm \frac{tg(\lambda_2 - L_2)}{\cos . \psi} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (6)$$

NOTE.—$\delta > 90^\circ$ when $(\lambda_2 - L_2) \begin{matrix} < 90^\circ \\ > 270^\circ \end{matrix}$; $\delta < 90^\circ$ when $(\lambda_2 - L_2) > 90^\circ < 270^\circ$.

δ positive and $< 180^\circ$.

$$tg\tau = \frac{D}{R_2 . \sin(\lambda_3 - \lambda_1)} \quad \tau \text{ positive and } < 180^\circ \quad \dots \quad \dots \quad (7)$$

$$T = \frac{D}{\sin \tau} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (8)$$

$$tg\Sigma = \pm \frac{C}{T . \sin(\tau + \psi)} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (9)$$

$$\sigma = \Sigma - \delta \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (10)$$

NOTE.—σ always $< 90^\circ$, positive or negative. Take Σ positive or negative, so that σ has always its least value :

$$S = \frac{C . \sin \delta}{\sin \Sigma} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (11)$$

$$P = \frac{t_1}{t_2} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

$$d = \frac{P+1}{A+PB} S \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (13)$$

$$tgu = \frac{1}{d \,.\, \cos \,.\, \sigma}\ u \text{ positive and } < 180^\circ \ldots \quad \ldots \quad \ldots \quad (14)$$

$$tg\omega = \frac{tg\sigma \,.\, \cos u}{\sqrt{2} \,.\, \sin (45^\circ - u)} \text{ comp. log } \sqrt{2} = 9{\cdot}84949 - 10 \quad (15)$$

NOTE.—ω is always $< 90^\circ$, positive or negative.

$$\Omega = \frac{d \,.\, \sin \sigma}{\sin \omega} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

$$h = \Omega(A + PB)R_2^3 \sin^3 \delta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

$$j_1 = \frac{k^2}{6} t_1 (t_1 + 2t_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

$$j_2 = \frac{k^2}{6} t_2 (2t_1 + t_2) \log \frac{k^2}{6} = 5{\cdot}69301 - 10 \quad \ldots \quad \ldots \quad (19)$$

$$\mathfrak{M} = \frac{Aj_1 + PBj_2}{h} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

3. *Solution of* $\mathfrak{M} \,.\, sin^4 z = \sin (z + \omega)$.

4. *Calculation of* r_1 *and* r_3.—

$$r_2 = \frac{R_2 \sin \delta}{\sin z} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

$$r_2 = \frac{R_2 \,.\, \cos \beta_2 \,.\, \sin (\delta - z)}{\sin z} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

$$Y_1 = \frac{j_1}{r_2^3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

$$Y_2 = \frac{j_2}{r_2^3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

$$n_1 = \frac{t_2}{t_1 + t_2} (1 + Y_1) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

$$n_2 = \frac{t_1}{t_1 + t_2} (1 + Y_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

$$r_1 = r_2 \,.\, \sin (\lambda_2 - \lambda_3) + n_1 R_1 \sin (L_1 - \lambda_3) + n_2 R_3 \sin (L_3 - \lambda_3) - R_2 \sin (L_2 - \lambda_3) \Big\} : n_1 \sin (\lambda_1 - \lambda_3) \quad (27)$$

$$r_3 = \frac{r_2 tg\beta_2 - n_1 r_1 tg\beta_1}{n_2 tg\beta_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

$$ctg\delta_1 = \frac{r_1}{R_1 \sin (\lambda_1 - L_1)} - \cot (\lambda_1 - L_1) \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

$$l_1 = \lambda_1 + \delta_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

$$ctg\delta_3 = \frac{r_3}{R_3 \sin(\lambda_3 - L_3)} - \cot(\lambda_3 - L_3) \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

$$l_3 = \lambda_3 + \delta_3 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

$$tg\, b_1 = \frac{r_1\, tg\beta_1 \sin(l_1 - \lambda_1)}{R_1 \sin(\lambda_1 - L_1)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

$$tg b_3 = \frac{r_3 tg\beta_3 \sin(l_3 - \lambda_3)}{R_3 \sin(\lambda_3 - L_3)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

$$r_1 = \frac{r_1\, tg\beta_1}{\sin b_1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

$$r_3 = \frac{r_3\, tg\beta_3}{\sin b_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

5. *Calculation of Inclination and Ascending Node.*—

$$tg\, D = \frac{tgb_1 \,.\, \sin(l_3 - l_1)}{tgb_3 - tgb_1 \,.\, \cos(l_3 - l_1)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

$$\Omega = l_1 - D \quad D < 180° \text{ or } > 180° \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

Note whether the motion is direct or retrograde.

$$tg i = \frac{tgb_1}{\sin D} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

For direct motion $i < 90°$; for retrograde $i > 90°$.

6. *Calculation of Eccentricity and Major Axis.*—

$$\sin(v_3 \sim v_1) = \frac{\sin(l_3 \sim l_1) \,.\, \cos b_1 \,.\, \cos b_3}{\cos i} \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

$$\sqrt{p'} = \frac{r_1 r_3 \sin(v_3 \sim v_1)}{k(t_1 + t_2)} \quad \log k = \bar{2}{\cdot}23558 \quad \ldots \quad \ldots \quad (41)$$

$$\sqrt{p} = \left[1 + \frac{\sin^2(v_3 \sim v_1) \,.\, \sqrt{r_1 r_3}}{6p'}\right] \sqrt{p'} \quad \ldots \quad \ldots \quad (42)$$

$$tg v_1 = \cot(v_3 - v_1) - \frac{r_1(p - r_3)}{r_3(p - r_1)\sin(v_3 - v_1)} \quad \ldots \quad \ldots \quad (43)$$

$$e^2 = \frac{(p - r_1)(p - r_3)}{r_1 r_3 \cos v_1 \,.\, \cos v_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

$$a = \frac{p}{1 - e^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

$$T^d = a^{\frac{3}{2}} \times 365{\cdot}256^d \qquad \log 365{\cdot}256 = 2{\cdot}56260 \quad (46)$$

$$m'' = \frac{1296\,000''}{T^d} \qquad \log 1296\,000 = 6{\cdot}11260 \quad (47)$$

7. *Calculation of the Longitude of Perihelium and of Mean Longitude.—*

$$tgu_1 = \frac{tgD}{\cos i} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

When $D < 180°$ $u_1 < 180°$; when $D > 180°$ $u_1 > 180°$.

$$\Pi = u_1 + 8 - v_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (49)$$

$$tg\frac{w}{2} = tg\frac{v_1}{2}\sqrt{\frac{1-e}{1+e}} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (50)$$

$$M = w - e \cdot \sin w \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (51)$$

M and w to be taken in parts of the radius.

$$\tau^d = E_1 - E_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

$$L'' = \tau^d m'' + M \times 206\,265''$$

$$\log 206\,265 = 5{\cdot}31442 \quad \ldots \quad \ldots \quad (53)$$

$$L_0 = L + \Pi \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (54)$$

Calculation of Juno's Orbit.

1. *The Data.—*

$$t_1 = 11^d{\cdot}96324 \qquad t_2 = 9^d{\cdot}97119$$

$$\log R_1 = 9{\cdot}99968 \qquad \log R_2 = 9{\cdot}99810 \qquad \log R_3 = 9{\cdot}99697$$

$$L_1 = 192° 28' 28'' \qquad L_2 = 204° 19' 49'' \qquad L_3 = 214° 16' 10''$$

$$\lambda_1 = 354° 44' 32'' \qquad \lambda_2 = 352° 34' 22'' \qquad \lambda_3 = 351° 34' 30''$$

$$\beta_1 = -4° 59' 31'' \qquad \beta_2 = -6° 21' 55'' \qquad \beta_3 = -7° 17' 51''$$

2. *Preparatory Calculations.—*

$$\log P = 0{\cdot}07910 \qquad t_1 + t_2 = 21^d{\cdot}93443$$

$$\log t_1 = 1{\cdot}07785 \qquad \log t_2 = 0{\cdot}99875 \qquad \log (t_1 + t_2) = 1{\cdot}34113$$

$$\log (t_1 + 2t_2) = 1{\cdot}50387 \qquad \log (2t_1 + t_2) = 1{\cdot}53017$$

$$\lambda_1 - L_1 = 162° 16' 4'' \qquad \lambda_1 - L_2 = 150° 24' 43''$$

$$\lambda_1 - L_3 = 140° 28' 22''$$

$$\lambda_3 - L_1 = 159° 6' 2'' \qquad \lambda_3 - L_2 = 147° 14' 41''$$

$$\lambda_3 - L_3 = 137° 18' 20''$$

$$\lambda_3 - \lambda_1 = -3° 10' 2'' \qquad \lambda_3 - \lambda_2 = -59' 52''$$

$$\lambda_2 - \lambda_1 = -2° 10' 10'' \qquad \lambda_2 - L_2 = 148° 14' 33''$$

$$L_3 - L_2 = 9° 56' 21'' \qquad L_3 - L_1 = 21° 47' 42''$$

$$L_2 - L_1 = 11° 51' 21''$$

3. *Preliminary Calculations for* A, B, C, *and* D.—

$\log tg\beta_3$	9·10741 (*n*)	$\log tg\beta_3$	9·10741 (*n*)	$\log tg\beta_3$	9·10741 (*n*)
$\log \sin(\lambda_1-L_1)$	9·48368	$\log \sin(\lambda_1-L_3)$	9·80376	$\log \sin(\lambda_1-L_2)$	9·69351
$\log (s_1)$	8·59109 (*n*)	$\log (s_3)$	8·91117 (*n*)	$\log (s_5)$	8·80092 (*n*)
$(s_1) = -0·039003$		$(s_3) = -0·81502$		$(s_5) = -0·063230$	
$\log tg\beta_1$	8·94124 (*n*)	$\log tg\beta_1$	8·94124 (*n*)	$\log tg\beta_1$	8·94124 (*n*)
$\log \sin(\lambda_3-L_1)$	9·55234	$\log \sin(\lambda_3-L_3)$	9·83129	$\log \sin(\lambda_3-L_2)$	9·73324
$\log (s_2)$	8·49358 (*n*)	$\log (s_4)$	8·77253 (*n*)	$\log (s_6)$	8·67448 (*n*)
$(s_2) = -0·031159$		$(s_4) = -0·059229$		$(s_6) = -0·047259$	
$(s_1)-(s_2) = -0·007844$		$(s_3)-(s_4) = -0·022273$		$(s_5)-(s_6) = -0·015971$	
$\log tg\beta_1$	8·94124 (*n*)	$\log tg\beta_3$	9·10741 (*n*)	$(s_7) = 0·073457$	
$\log \cos(\lambda_3-L_2)$	9·92479 (*n*)	$\log \cos(\lambda_1-L_2)$	9·93932 (*n*)	$(s_8) = 0·111360$	
$\log (s_7)$	8·86603	$\log (s_8)$	9·04673	$(s_4)-(s_8) = -0·037903$	

4. *Calculation of* A, B, C, *and* D. (1) *to* (4).—

$\log R_1$	9·99968	$\log R_3$	9·99697	$\log R_2$	9·99810	$\log R_2$	9·99810
$\log [(s_1)-(s_2)]$	7·89454 (*n*)	$\log [(s_3)-(s_4)]$	8·34778 (*n*)	$\log [(s_5)-(s_6)]$	8·20333 (*n*)	$\log [(s_7)-(s_8)]$	8·57867 (*n*)
$\log A$	7·89422 (*n*)	$\log B$	8·34475 (*n*)	$\log C$	8·20143 (*n*)	$\log D$	8·57677 (*n*)
		$\log P$	0·07910		$A = -0·0078382$		
		$\log PB$	8·42385 (*n*)		$PB = -0·0265370$		
		$\log (A+PB) = 8·53624$ (*n*)			$A+PB = -0·0343752$		

5. *Calculations preceding the Equation of Gauss.* (5) *to* (20).—

log $tg\beta_2$	9·04748 (*n*)	log $tg(\lambda_2-L_2)$	9·79169 (*n*)	og D	8·57677 (*n*)
log sin (λ_2-L_2)	9·72125	log cos ψ	9·99046 (*n*)	c. log R_2	0·00190
log $tg\psi$	9·32623 (*n*)	log $tg\delta$	9·80123	c. log sin $(\lambda_3-\lambda_1)$	1·25766 (*n*)
$\psi = 168°\ 2'$		$\delta = 32°\ 19'\ 24''$		$r = 34°\ 27'\ 2''$	log $tg r$ 9·83633

log D	8·57677 (*n*)	log C	8·20143 (*n*)	$\Sigma =$	31° 56′ 10″
log sin r	9·75259	c. log T	1·17582 (*n*)	$\delta =$	32° 19′ 24″
log T	8·82418 (*n*)	c. log sin $(r+\psi)$	0·41746 (*n*)	$\sigma =$	− 23′ 14″
$r+\psi = 202°\ 29'\ 2''$		log $(tg\Sigma)$	9·79471 (*n*)	P+1 =	2·1998
log C	8·20143 (*n*)	log S	8·20611 (*n*)	log d	0·01225
log sin δ	9·72811	log (P+1)	0·34238	log cos σ	9·99999
c. log sin Σ	0·27657	c. log (A+PB)	1·46376 (*n*)	log cot u	0·01224
log S	8·20611 (*n*)	log d	0·01225	$u =$	44° 11′ 34″
log $tg\sigma$	7·82984 (*n*)	log d	0·01225	$45°-u =$	48′ 26″
log cos u	9·85552	log sin σ	7·82983 (*n*)	log Ω	8·46858
c. log $\sqrt{2}$	9·84949	c. log sin ω	0·62650 (*n*)	log (A+PB)	8·53624 (*n*)
c. log sin $(45°-u)$	1·85114	log Ω	8·46858	log R_2^3	9·99430
log $tg\omega$	9·38599 (*n*)	$\omega = -13°\ 40'\ 11''$		log sin$^3\delta$	9·18433
				log h	6·18345 (*n*)

	$\log \frac{k_2}{6}$	5·69301−10		$\log \frac{k^2}{6}$	5·69301−10
	$\log t_1$	1·07785		$\log t_2$	0·99875
	$\log (t_1+2t_2)$	1·50387		$\log (2t_1+t_2)$	1·53017
	$\log j_1$	8·27473−10		$\log j_2$	8·22193−10
log A	7·89422 (*n*)	log PB	8·42385 (*n*)	(s_9)	$=-0\cdot00014755$
$\log j_1$	8·27473	$\log j_2$	8·22193	(s_{10})	$=-0\cdot00044236$
$\log (s_9)$	6·16895 (*n*)	$\log (s_{10})$	6·64578 (*n*)	$(s_9)+(s_{10})$	$=-0\cdot00058991$
		$\log [(s_9)+(s_{10})]$	6·77079 (*n*)		
		$\log h$	6·18345 (*n*)		
		$\log \mathfrak{M}$	0·58734		

6. *Solution of the Equation of Gauss.*—

$\log \sin \omega$	9·374	$\lambda =$	0·5	$\log \sin z_2$	9·394
	4				4
$\log \sin^4 \omega$	7·496	$\mu =$	10·5	$\log \sin^4 z_2$	7·576
$\log \mathfrak{M}$	0·587	$\mu-4\lambda =$	8·5	Δ	
				$\frac{\Delta}{\mu-4\lambda} =$	9′ 24″
$\log \sin (z_2+\omega)$	8·083	$\Delta =$	80		
$z_2+\omega =$	41′ 42″	$\omega =$	−13° 40′ 11″	$z_2 =$	14° 21′ 53″
				$z_2 =$	14° 31′ 17″

log sin z_3	9·39923	$\lambda = 0{\cdot}8$	log sin z_4 9·39993
	4		4
log sin⁴ z_3	7·59692	$\mu = 14$	log sin⁴ z 7·59972
log $\mathfrak{M}$	0·58734	$\mu - 4\lambda = 10{\cdot}8$	Δ
log sin $(z_4+\omega)$	8·18426	$\Delta = 280$	$\overline{\mu-4\lambda} = 26''$
$z_4+\omega = 5^\circ\ 2'\ 33''$		$\omega = -13^\circ\ 40'\ 11''$	$z_4 = 14^\circ\ 32'\ 44''$
Test.			$z_5 = 14^\circ\ 33'\ 10''$
log sin z_5	9·40014	$\lambda = 0{\cdot}8$	log sin z_6 9·40014
	4		4
log sin⁴ z_5	7·60056	$\mu = 13{\cdot}7$	log sin⁴ z_6 7·60056
log $\mathfrak{M}$	0·58734	$\mu - 4\lambda = 10{\cdot}5$	Δ
log sin $(z_6+\omega)$	8·18790	$\Delta = 0$	$\overline{\mu-4\lambda} = 0''$
$z_6+\omega = 5^\circ\ 2'\ 59''$		$\omega = -13^\circ\ 40'\ 11'$	$z_6 = 14^\circ\ 33'\ 10'$

7. *Calculation of* r_1 *and* r_3. (21) *to* (28).—

log R_2	9·99810	$\delta = 32^\circ\ 19'\ 24''$	log R_2 9·99810
log sin δ	9·72811	$z = 14\ 33\ 10$	log cos β_2 9·99731
c. log sin z	0·59986	$\delta - z = 17^\circ\ 46'\ 14''$	c. log sin z 0·59986
log r_2	0·32607	$r_2 = 2{\cdot}1187$	log sin $(\delta-z)$ 9·48459
$1+Y_1 = 1{\cdot}001979$		$1+Y_2 = 1{\cdot}001753$	log r_2 0·07986

$\log j_1$	8·27473	$\log t_2$	0·99875
$\log r_2^3$	0·97821	c. $\log (t_1+t_2)$	8·65887
$\log Y_1$	7·29652	$\log (1+Y_1)$	0·00086
$Y_1 = 0·001979$		$\log n_1$	9·65848

$\log r_2$	0·07986	
$\log \sin (\lambda_2-\lambda_3)$	8·24090	
$\log (s_n)$	8·32076	
$\log n_1$	9·65848	
$\log R_1$	9·99968	
$\log \sin (L_1-\lambda_3)$	9·55234	(n)
$\log (s_{12})$	9·21050	(n)
$(s_{11}) =$	0·020929	
$(s_{12}) =$	−0·162366	
$(s_{13}) =$	−0·367908	
$(s_{14}) =$	0·538688	
$(S) =$	0·029343	

$\log r_2$	0·07986	$\log n_1$	9·65848
$\log tg\beta_2$	9·04748 (n)	$\log r_1$	0·06668
$\log (s_{15})$	9·12734 (n)	$\log tgB_1$	8·94124 (n)
$\log (S_1)$	8·94294 (n)	$\log (s_{16})$	8·66640 (n)

$\log j_2$	8·22193	$\log t_1$	1·07785
$\log r^3_2$	0·97821	c. $\log (t_1+t^2)$	8·65887
$\log Y_2$	7·24372	$\log (1+Y_2)$	0·00076
$Y_2 = 0·001753$		$\log n_2$	9·73748

$\log R_2$	9·99810	
$\log \sin (L_2-\lambda_3)$	9·73324	(n)
$\log (s_{14})$	9·73134	(n)
$\log (n_2)$	9·73748	
$\log R_3$	9·99697	
$\log \sin (L_3-\lambda_3)$	9·83129	(n)
$\log (s_{13})$	9·56574	(n)
$\log (S)$	8·46750	
c. $\log n_1$	0·34152	
c. $\log \sin (\lambda_1-\lambda_3)$	1·25766	
$\log r_1$	0·06668	

$(s_{15}) =$	−0·134074
$-(s_{16}) =$	0·046387
$(S_1) =$	−0·087687

c. log n_2	0·26252	l_1	$= 362° 55' 13''$	l_3	$= 370° 20' 12''$
c. log $tg\beta_3$	0·89259 (n)	λ_1	$= 354° 44' 32''$	λ_3	$= 351° 34' 30''$
log r_3	0·09805	$l_1-\lambda_1 =$	$8° 10' 41''$	$l_3-\lambda_3 =$	$18° 45' 42''$

8. *Calculation of Heliocentric Coordinates.* (29) *to* (36).—

	log r_1	0·06668			log r_3	0·09805	
	c. log R_1	0·00032			c. log R_3	0·00303	
	c. log sin (λ_1-L_1)	0·51632			c. log sin (λ_3-L_3)	0·16871	
	log (s_{17})	0·58332			log (s_{18})	0·26979	
(s_{17})	3·83109	$\lambda_1 = 354° 44' 32''$		(s_{18})	1·86117	$\lambda_3 = 351° 34' 30''$	
$-\cot(\lambda_1-L_1)$	3·12734	$\delta_1 = 8° 10' 41''$		$-\cot(\lambda_3-L_3)$	1·08390	$\delta_3 = 18° 45' 42''$	
cot δ_1	6·95843	$l_1 = 2° 55' 13''$		cot δ_3	2·94507	$l_3 = 10° 20' 12''$	
log r_1	0·06668	log r_1	0·06668	log r_3	0·09805	log r_3	0·09805
log $tg\beta_1$	8·94124 (n)	log $tg\beta_1$	8·94124 (n)	log $tg\beta_3$	9·10741 (n)	log $tg\beta_3$	9·10741
log sin $(l_1-\lambda_1)$	9·15304	c. log sin b_1	1·32292 (n)	log sin $(l_3-\lambda_3)$	9·50736	c. log sin b_3	1·11665(n)
c. log R_1	0·00032	log r_1	0·33084	c. log R_3	0·00303	log r_3	0·32211
c. log sin (λ_1-L_1)	0·51632			c. log sin (λ_3-L_3)	0·16871		
	8·67760				8·88456		
log tgb_1	(n) $b_1 = -2° 43' 30''$			log tgb_3		(n) $b_3 = -4° 23' 3''$	

Motion direct. $i < 90°$.

9. *Calculation of Inclination and Ascending Node.* (37) *to* (39).—

$ltgb_1$	8·67760(*n*)	$tgb_3 = -0{\cdot}076665$	$l_3 =$	10° 20′ 12″	
$l \cos (l_3 - l_1)$	9·99635	$-(s_{18}) =$	0·047201	$l_1 =$	2 55 13
$l(s_{18})$	8·67395(*n*)	$(s_{19}) = -0{\cdot}029464$		$l_3 - l_1 =$	7 24 59

$ltgb_1$	8·67760(*n*)	$ltgb_1$	8·67760(*n*)	$l_1 =$	2° 55′ 13″ + 360°
$l \sin (l_3 - l_1)$	9·11085	$l \sin$ D	9·30993(*n*)	D =	11 46 44 + 180
c. $l(s_{19})$	1·53071(*n*)	$ltgi$	9·36767	☊ =	171 8 29
ltgD	9·31916	$i =$	13° 7′ 30″		

10. *Calculation of Eccentricity and Major Axis.* (40) *to* (47).—

log sin $(l_3 \sim l_1)$	9·11085	log sin $(v_3 \sim v_1)$	9·12058
log cos b_1	9·99951	log r_1	0·33084
log cos b_3	9·99873	log r_3	0·32211
c. log cos i	0·01149	c. log k	1·76442
log sin $(v_3 \sim v_1)$	9·12058	c. log $(t_1 + t_2)$	8·65887
log p'	0·39364	log $\sqrt{p'}$	0·19682
log 6	0·77815	$v_3 \sim v_1 =$	7° 35′ 7″
log 6p'	1·17169		

log sin² $(v_3 \sim v_1)$	8·24116			log $(1 + (s_{20}))$	0·00108
log $\sqrt{r_1}$	0·16542			log $\sqrt{p'}$	0·19682
log $\sqrt{r_3}$	0·16105			log $\sqrt{p}$	0·19790
c. log 6p'	8·82831	$p =$	2·48768	log p	0·39580
log (s_{20})	7·39594	$(s_{20}) =$	0·0024885	$1 + (s_{20}) =$	1·00249

$p =$	2·48768	$p =$	2·48768
$r_1 =$	2·14210	$r_3 =$	2·09948
$p - r_1 =$	0·34558	$p - r_3 =$	0·38820
log $(p - r_3)$	9·58906		
log r_1	0·33084	$-(s_{21}) =$	−8·68280
c. log r_3	9·67789	cot $(v_2 - v_1) =$	7·52104
c. log $(p - r_1)$	0·46145	$tg\ v_1 =$	−1·16176

c. log sin (u_3-u_1)	0·87942	$v_1 = -49\ 16\ 46$	
log (s_{21})	0·93866	$\frac{v_1}{2} = -24\ 38\ 23$	

log $(p-r_1)$	9·53855	$v_1 = -49\ 16\ 46$	
log $(p-r_3)$	9·58906	$v_3-v_1 = 7\ 35\ 7$	
c. log r_1	9·66916	$u_3 = -41\ 41\ 39$	
c. log r_3	9·67789	log p	0·39580
c. log cos v_1	0·18542	l. cos$^2\phi$	9·97256
c. log cos v_3	0·12686	log a	0·42324
log e^2	8·78694	$e = 0{\cdot}24744$	
log $e = l$ sin $\phi = 9{\cdot}39347$		$\phi = 14°\ 19'\ 33''$	

log $a^{\frac{3}{2}}$	0·63486	log 1·296000	6·11260
log $365^{d}{\cdot}256$	2·56260	log T^d	3·19746
log T^d	3·19746	log m'' 2·91514	
$T^d = 1575^{d}{\cdot}65$		$m'' = 822''{\cdot}5$	

11. *Calculation of Longitude of Perihelium and Mean Longitude.* (48) *to* (54).—

log *tg*D	9·31916	$u_1 = 192°\ 5'\ 8''$	
log cos i	9·98851	$\Omega = 171\ 8\ 29$	
log tgu_1	9·33065	363 13 37	
$1+e = 1{\cdot}24744$		$-v_1 = 49\ 16\ 46$	
$1-e = 0{\cdot}75256$		$\Pi = 52\ 30\ 23$	

log $tg\frac{v}{2}$	9·66151(n)	log e 9·39347	$w = -0{\cdot}684489$		
log $\sqrt{1-e}$	9·93827	log sin w 9·80091(n)	$-(s_{22}) =$	0·156452	
c. log $\sqrt{1+e}$	9·95199	log (s_{22}) 9·19438(n)	M $= -0{\cdot}528037$		
log $tg\frac{w}{2}$	9·55177(n)	$\frac{w}{2} = -19°\ 36'\ 33''$	$w = -39°\ 13'\ 6''$		

$E_1 = 1804$, Oct. $5{\cdot}458644$ log M.	$9{\cdot}72263(n)$
$E_0 = 1805$, Jan. $0{\cdot}000000$ log $206265''$	$5{\cdot}31442$
$\tau^d =$ $86^d{\cdot}541356$ log$(s_{23}$	$5{\cdot}03705(n)$

log τ^d	$1{\cdot}93722$	$(s_{23}) = -10''8905$	L $= -10^\circ\ 28'\ 45''$
log m''	$2{\cdot}91514$	$(s_{24}) = 71180$	II $= 52\ 30\ 23$
log (s_{24})	$4{\cdot}85236$	L$'' = -37725$	$L_0 = 41\ 58\ 22$

Syra, Greece: 1903 *August* 20.

Ephemeris for Physical Observations of Saturn, 1903–4.

By A. C. D. Crommelin.

The discovery by Professor Barnard of a conspicuous white spot on *Saturn* last July has reawakened interest in the question of the planet's rotation, and has illustrated the remarkable variation of rate that prevails in different latitudes. At Mr. Denning's suggestion I have computed an ephemeris giving the longitudes of central meridian and the times of transit of the zero meridian on two assumptions as to the rotation period, viz.:

System I. $10^h\ 14^m{\cdot}4$ for equatorial spots (Hall).

System II. $10^h\ 37^m{\cdot}92$ for the spot discovered by Barnard last July (period provisionally deduced by Denning).

The quantities corresponding to P, B, *B* in the *Jupiter* ephemeris may be taken from the *Nautical Almanac* ephemeris of the ring, or more conveniently from the *Connaissance des Temps*, as this is fuller and contains the longitude of the Earth in the plane of the ring. In order to utilise the data of the *Connaissance* without interpolation, I give the longitude of the central meridian for Paris midnight, corresponding to $11^h\ 50^m{\cdot}65$ G.M.T.; but the transits of zero meridian are given for G.M.T. The quantities are to be interpolated for the time for which they are required, the equation of light having been already applied. The approximate increase of the longitude of central meridian in five days is 4219° in System I., 4063° in System II. I have commenced a new system of zero meridians, as Mr. Marth used a different period of rotation ($10^h\ 13^m{\cdot}6$) in his ephemerides, of which the last appeared in vol. lv. p. 164. The small figures between the transits of zero meridians denote the number of intervening rotations, and the table at the end facilitates the determination of the times of intervening transits.

Paris Midnight. 1903.	Light Time.	Longitude of Central Meridian. I. 843°·750.	Longitude of Central Meridian. II. 812°·641.	G.M.T. of Preceding Transit of Zero Meridian. System I.		G.M.T. of Preceding Transit of Zero Meridian. System II.	
	m	°	°	h m		h m	
June 8	77·49	0·00	0·00	1 36·3	12	1 12·8	12
13		259·23	103·69	4 28·3	12	8 46·9	11
18	76·45	158·47	207·36	7 20·2	12	5 43·2	11
23		57·70	311·01	10 12·2	11	2 39·6	12
28	75·63	316·94	54·66	2 49·8	12	10 13·8	11
July 3		216·17	158·33	5 41·8	12	7 10·1	11
8	75·01	115·39	262·01	8 33·7	12	4 6·4	12
13		14·60	5·68	11 25·7	11	11 40·7	11
18	74·59	273·81	109·33	4 3·4	12	8 37·0	11
23		172·98	212·94	6 55·5	12	5 33·4	11
28	74·44	72·11	316·53	9 47·6	11	2 29·8	12
Aug. 2		331·21	60·09	2 25·6	12	10 4·2	11
7	74·52	230·28	163·61	5 17·7	12	7 0·8	11
12		129·31	267·10	8 10·0	12	3 57·4	12
17	74·85	28·28	10·53	11 2·4	11	11 32·0	11
22		287·20	113·92	3 40·5	12	8 28·8	11
27	75·43	186·07	217·24	6 33·1	12	5 25·7	11
Sept. 1		84·89	320·53	9 25·8	11	2 22·7	12
6	76·21	343·64	63·74	2 4·1	12	9 57·7	11
11		242·34	166·91	4 57·0	12	6 54·9	11
16	77·18	140·98	270·01	7 50·0	12	3 52·2	12
21		39·57	13·07	10 43·1	11	11 27·5	11
26	78·29	298·10	116·06	3 22·0	12	8 25·0	11
Oct. 1		196·56	218·98	6 15·2	12	5 22·6	11
6	79·54	94·96	321·85	9 8·5	11	2 20·3	12
11		353·31	64·67	1 47·6	12	9 56·1	11
16	80·87	251·62	167·45	4 41·1	12	6 53·9	11
21		149·88	270·18	7 34·7	12	3 51·8	12
26	82·24	48·10	12·87	10 28·5	11	11 27·8	11
31		306·27	115·52	3 7·9	12	8 26·0	11
Nov. 5	83·62	204·41	218·12	6 1·8	12	5 24·2	11
10		102·51	320·69	8 55·7	12	2 22·4	12
15	84·96	0·56	63·23	11 49·7	11	9 58·7	11
20		258·61	165·75	4 29·2	12	6 56·9	11
25	86·22	156·64	268·25	7 23·2	12	3 55·3	12
30		54·66	10·74	10 17·3	11	11 31·6	11
Dec. 5	87·38	312·66	113·21	2 57·0	12	8 30·1	11
10		210·66	215·67	5 51·1	12	5 28·5	11

Paris Midnight.	Light Time.	Longitude of Central Meridian. I. 843°·750.	II. 812°·641.	G.M.T. of Preceding Transit of Zero Meridian. System I.		System II.	
1903	m	°	°	h m		h m	
Dec. 15	88·41	108·65	318·13	8 45·2	12	2 26·9	12
20		6·65	60·60	11 39·3	11	10 3·3	11
25	89·27	264·65	163·06	4 18·9		7 1·7	
1904.							
May 13	81·68	301·20	164·29	3 16·5	12	6 59·5	11
18		200·30	267·77	6 8·8	12	3 56·2	12
23	80·30	99·36	11·27	9 1·1	11	11 30·7	11
28		358·45	114·80	1 38·9	12	8 27·2	11
June 2	78·98	257·57	218·36	4 31·1	12	5 23·8	11
7		156·72	321·96	7 23·2	12	2 20·2	12
12	77·72	55·91	65·58	10 15·2	11	9 54·5	11
17		315·10	168·22	2 52·9	12	6 50·8	11
22	76·60	214·30	272·86	5 44·9	12	3 47·2	12
27		113·52	16·52	8 36·9	12	11 21·4	11
July 2	75·62	12·74	120·19	11 28·9	11	8 17·7	11
7		271·97	223·87	4 6·5	12	5 14·0	11
12	74·84	171·20	327·54	6 58·5	12	2 10·3	12
17		70·41	71·20	9 50·5	11	9 44·5	11
22	74·27	329·62	174·86	2 28·1	12	6 40·8	11
27		228·82	278·51	5 20·2	12	3 37·2	12
Aug. 1	73·91	127·99	22·13	8 12·2	12	11 11·4	11
6		27·13	125·72	11 4·3	11	8 7·9	11
11	73·82	286·23	229·28	3 42·2	12	5 4·4	11
16		185·30	332·81	6 34·4	12	2 1·0	12
21	73·97	84·33	76·30	9 26·7	11	9 35·5	11
26		343·32	179·75	2 4·7	12	6 32·2	11
31	74·36	242·26	283·15	4 57·2	12	3 28·9	12
Sept. 5		141·15	26·50	7 49·8	12	11 3·7	11
10	74·99	39·98	129·79	10 42·4	11	8 0·7	11
15		298·75	233·03	3 20·8	12	4 57·7	11
20	75·82	197·47	336·21	6 13·6	12	1 54·9	12
25		96·12	79·33	9 6·6	11	9 30·1	11
30	76·84	354·71	182·39	1 45·3	12	6 27·5	11
Oct. 5		253·25	285·40	4 38·4	12	3 24·9	12
10	78·00	151·74	28·36	7 31·7	12	11 0·4	11
15		50·16	131·26	10 25·0	11	7 58·0	11
20	79·29	308·54	234·10	3 4·0	12	4 55·8	11
25		206·87	336·90	5 57·6	12	1 53·6	12
30	80·62	105·15	79·65	8 51·2	12	9 29·5	11

Paris Midnight. 1904.	Light Time.	Longitude of Central Meridian. I. 843°·750.	II. 812°·641.	G.M.T. of Preceding Transit of Zero Meridian. System I.		System II.	
	m	°	°	h m		h m	
Nov. 4		3·39	182·36	11 44·8	11	6 27·5	11
9	82·00	261·58	285·02	4 24·2	12	3 25·5	12
14		159·74	27·65	7 18·0	12	11 1·6	11
19	83·37	57·86	130·24	10 11·9	11	7 59·8	11
24		315·96	232·81	2 51·3	12	4 58·0	11
29	84·68	214·05	335·36	5 45·3	12	1 56·3	12
Dec. 4		112·10	77·88	8 39·3	12	9 32·6	11
9	85·92	10·13	180·38	11 33·4	11	6 30·9	11
14		268·15	282·87	4 12·9	12	3 29·3	12
19	87·04	166·16	25·35	7 7·0	12	11 5·7	11
24		64·17	127·83	10 1·1	11	8 4·1	11
29	88·01	322·18	230·31	2 40·7		5 2·5	

The following interpolation table gives the quantities to apply to any transit of the zero meridian in order to obtain the neighbouring transits :

Increase of Central Meridian in 5 days.		6 Rotations.	5 Rotations.	4 Rotations.	3 Rotations.	2 Rotations.	1 Rotation.
	°	h m	h m	h m	h m	h m	h m
System I.	4219·2	61 26·0	51 11·7	40 57·3	30 43·0	20 28·7	10 14·3
	4218·8	26·4	12·0	57·6	43·2	28·8	14·4
	4218·4	26·7	12·3	57·8	43·4	28·9	14·5
	4218·0	27·1	12·5	58·0	43·5	29·0	14·5
System II.	4063·7	63 47·1	53 9·2	42 31·4	31 53·5	21 15·7	10 37·8
	4063·3	47·4	9·5	31·6	53·7	15·8	37·9
	4062·9	47·8	9·8	31·9	53·9	15·9	38·0
	4062·5	48·2	10·2	32·1	54·1	16·1	38·0

The argument is, of course, found by taking the difference of consecutive values of "longitude of Central Meridian." Only the unit and first decimal figures of the difference need be considered.

55 *Ulundi Road, Blackheath, S.E.:*
1903 *December* 11.